Online Linear Learning

John Langford, Machine Learning the Future, February 27

To follow along:
git clone
git://github.com/JohnLangford/vowpal_wabbit.git
wget `http://hunch.net/~jl/rcv1.tar.gz`

Linear Learning

Features: a vector $x \in \mathbb{R}^n$
Label: $y \in \mathbb{R}$
Goal: Learn $w \in \mathbb{R}^n$ such that
$\hat{y}_w(x) = \sum_i w_i x_i$ is close to y .

Linear Learning

Features: a vector $x \in \mathbb{R}^n$
Label: $y \in \{-1, 1\}$
Goal: Learn $w \in \mathbb{R}^n$ such that
$\hat{y}_w(x) = \text{sign}(\sum_i w_i x_i) = y$.

Online Linear Learning

Start with $\forall i : \quad w_i = 0$
Repeatedly:

1. Get features $x \in \mathbb{R}^n$.
2. Make linear prediction $\hat{y}_w(x) = \sum_i w_i x_i$.
3. Observe label $y \in [0, 1]$.
4. Update weights so $\hat{y}_w(x)$ is closer to y.

Online Linear Learning

Start with $\forall i : \quad w_i = 0$

Repeatedly:

1. Get features $x \in \mathbb{R}^n$.
2. Make logistic prediction $\hat{y}_w(x) = \frac{1}{1+e^{-\sum_i w_i x_i}}$.
3. Observe label $y \in [0, 1]$.
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   Example: $w_i \leftarrow w_i + \eta(y - \hat{y})x_i$.

An Example: The RCV1 dataset

Pick whether a document is in category CCAT or not.

Dataset size:

781K examples

60M nonzero features

1.1G bytes

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command: time vw --sgd rcv1.train.txt -c

takes 1-3 seconds on my laptop.

Reasons for Online Learning

1. Fast convergence to a good predictor
2. It's RAM efficient. You need store only one example in RAM rather than all of them. $\Rightarrow$ Entirely new scales of data are possible.
3. Online Learning algorithm = Online Optimization Algorithm. Online Learning Algorithms $\Rightarrow$ the ability to solve entirely new categories of applications.
4. Online Learning = ability to deal with drifting distributions.

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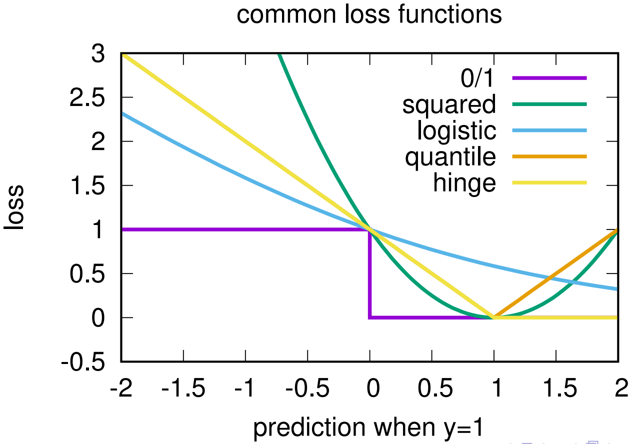


Know your loss function semantics

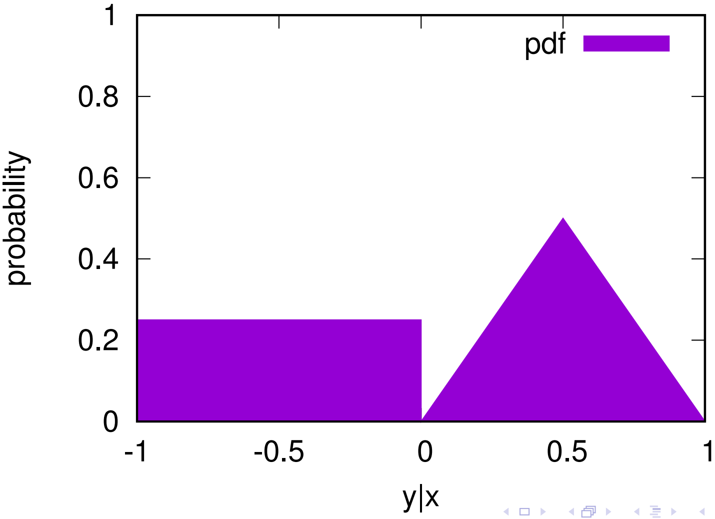
1. What is a typical price for a house?
2. What is the expected return on a stock?
3. What is the probability of a click on an ad?
4. Is the digit a 1?
5. What do you really care about?

Know your loss function semantics

1. What is a typical price for a house?
   quantile: minimizer = median
2. What is the expected return on a stock?
   squared: minimizer = expectation
3. What is the probability of a click on an ad?
   logistic: minimizer = probability
4. Is the digit a 1?
   hinge: closest 0/1 approximation
5. What do you really care about?
   often 0/1

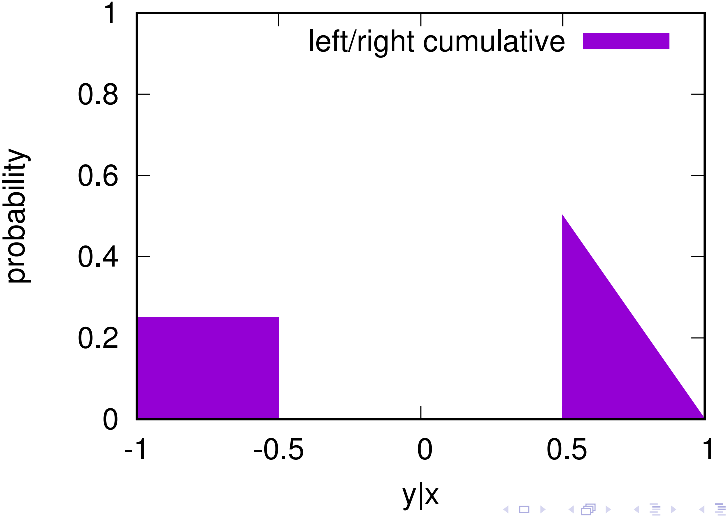
A proof for quantile regression

Consider conditional probability distribution $D(y|x)$.



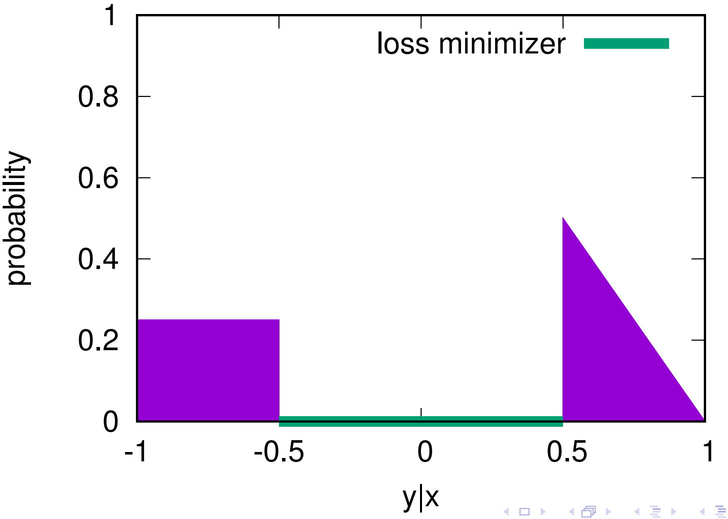
A proof for quantile regression

Consider equal mass tails. Where is loss minimized?



A proof for quantile regression

Minimizer is always between.



A proof for quantile regression

Works for any tails $\Rightarrow$ works for mass 0.5 tails.

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Progressive Validation

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Report loss $L = E_t l_t$.

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PV analysis

Let D be a distribution over x, y. Let
$\bar{l}_t = E_{(x,y)\sim D} L(\hat{y}_{w_t}(x), y)$
Theorem: For all probability distributions $D(x, y)$, for all online learning algorithms, with probability $1 - \delta$:

$$\left| L - E_t \bar{l}_t \right| \leq \sqrt{\frac{\ln 2/\delta}{2T}}$$

rcv1 with different loss functions

All the common loss functions are sound for binary classification, so which is best is an empirical choice.

```
vw --sgd rcv1.train.txt -c --loss_function
hinge --binary
vw --sgd rcv1.train.txt -c --loss_function
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Progressive validation often does not replace train/test discipline, but it can greatly aid empirical testing.

Part II, advanced updates

1. Importance weight invariance
2. Adaptive updates
3. Normalized updates

Learning with importance weights

A common scenario: you need to do classification but one choice is more expensive than the other.

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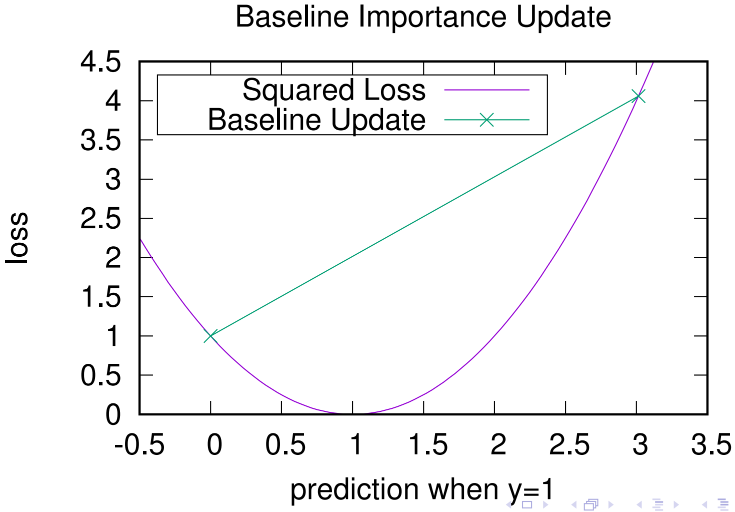
How do you modify the update to use I?

The baseline approach: $w_i \leftarrow w_i - \eta I \frac{\partial L(\hat{y}_w(x), y)}{\partial w_i}$.

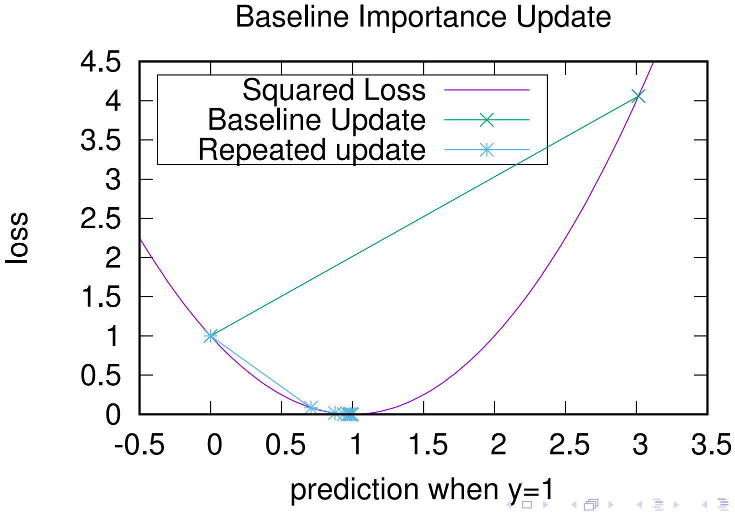
Dealing with the importance weights

$w_i \leftarrow w_i - \eta I \frac{\partial L(\hat{y}_w(x),y)}{\partial w_i}$ performs poorly.



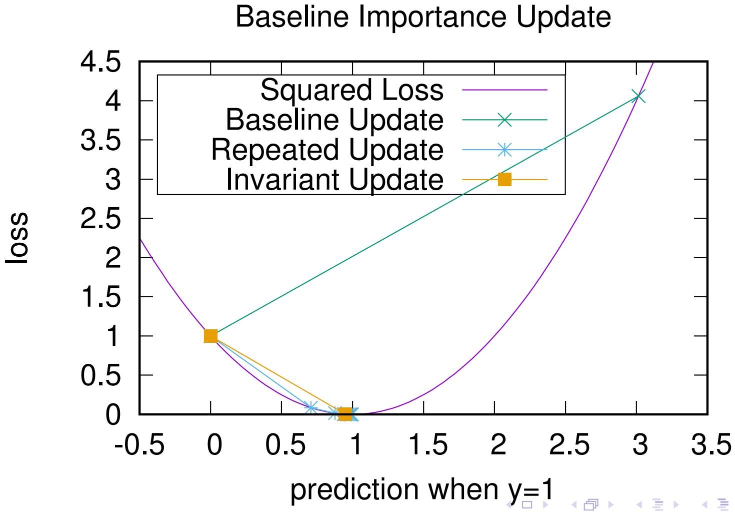
Dealing with the importance weights

A better approach: $w_i \leftarrow w_i - \eta \frac{\partial L(\hat{y}_w(x), y)}{\partial w_i}$ I times

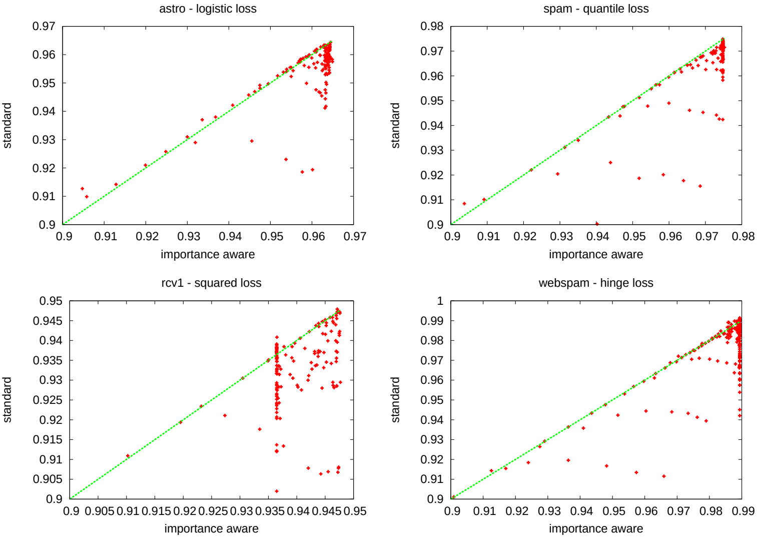


Dealing with the importance weights

An even better approach: $w_i \leftarrow w_i - s(\eta I)\frac{\partial L(\hat{y}_w(x),y)}{\partial w_i}$



Robust results for unweighted problems



rcv1 with an invariant update

`vw rcv1.train.txt -c --binary --invariant`

Performs slightly worse with the default learning rate, but much more robust to learning rate choice.

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Better answer: t, let $g_{it} = \frac{\partial L(\hat{y}_w(x_t), y_t)}{\partial w_i}$.
New update rule: $w_i \leftarrow w_{it} - \eta \frac{g_{it}}{\sqrt{\sum_{t'=1}^{t} g_{it'}^2}}$

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Common features stabilize quickly. Rare features can have large updates.

Adaptive Learning example

`vw rcv1.train.txt -c --binary --adaptive`

Slightly worse. Adding in --invariant -l 1 helps.

Dimensional Correction

g_{it} for squared loss $= 2(\hat{y}_w(x) - y)x_i$ so update is

$$w_i \leftarrow w_i - Cx_i$$

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The same form occurs for all linear updates.
Intrinsic problems! Doubling x_i implies halving w_i to get the same prediction.
$\Rightarrow$ Update rule has mixed units!

A standard solution: Gaussian sphering

For each feature x_i compute:
empirical mean $\mu_i = E_t x_{it}$
empirical standard deviation $\sigma_i = \sqrt{E_t(x_{it} - \mu_i)^2}$
Let $x_i' \leftarrow \frac{x_i - \mu_i}{\sigma_i}$.

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Problems:

1. Lose online.
2. RCV1 becomes a factor of 500 larger.

A scale-free update

NG(learning_rate η)

1. Initially $w_i = 0$, $s_i = 0$, $N = 0$
2. For each timestep t observe example (x, y)
    1. For each i, if $|x_i| > s_i$
        1. Renormalize w_i for new scale
        2. Adjust Scale
    2. $\hat{y} = \sum_i w_i x_i$
    3. Adjust global scale
    4. For each i,
        1. $w_i \leftarrow w_i - \eta$ (scale adjustment) $\frac{\partial L(\hat{y}, y)}{\partial w_i}$

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    3. $N \leftarrow N + \sum_i \frac{x_i^2}{s_i^2}$
    4. For each i,
        1. $w_i \leftarrow w_i - \eta \sqrt{\frac{t}{N}} \frac{1}{s_i^2} \frac{\partial L(\hat{y}, y)}{\partial w_i}$

In combination

An adaptive, scale-free, importance invariant update rule.

`vw rcv1.train.txt -c --binary`

Not the End

... there are many more problems with gradient descent. How do you fix them?

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