Tutorial on Machine Learning Reductions

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Scenario 1

You work for a charity as a spam optimizer.

Question: Who should you bother to ask money from?

"Oh look, this is binary classification. I'll just apply a decision tree to predict who provides money."

Result: $0.00 income on test set.

Oops: Mailing everyone $\Rightarrow$ $10000 income. You are fired.

Scenario 2

You work for a doctor predicting "cancer or not" given symptoms.

You choose to use a support vector machine.

But the doctor doesn't want a decision—a probability is preferred.

...so you return the "margin" as a probability.

Your probabilities are always near 0.5. You are fired.

Where did the Hollywood ending go?

Basic difficulty: World's problem is not problem solved by algorithm.

Solutions:

1. Design new algorithms! (The research employment act.)

2. Discover how to reuse old algorithms.

Learning reductions are the mathematics of #2.

Basic question: Can #2 do everything that #1 can?

Characteristics of Learning Reductions

1. Reductionist.

2. Elemental.

3. Works well in practice.

4. Easy.

It's reductionist (= good research direction)

Reductionist = cut problems into small problems, solve small problems, and compose to solve big problem.

Some other reductionist things:

1. The transistor for computations
2. Rendering triangles for rendering scenes
3. Much of science

Elemental

Reduction needs to know nothing about oracle learning algorithm except its type $\Rightarrow$Modularity, code reuse, universality.

1. Can reuse old learning algorithms

2. Can reuse old code

It's easy (= you can use it too)

The reductions method to solving learning problems:

1. Identify the type of learning problem B.

2. Find premade reduction R and oracle learning algorithm A.

3. Build a B predictor using $R^A + \text{data}$.

Given a Binary classifier, how can we solve

1. Importance Weighted Classification?

2. Class Probability estimation?

3. Multiclass Classification?

4. Cost Sensitive Classification?

5. Reinforcement Learning?

Classification Definition

- Problem: A measure D on $X \times \{0,1\}$ where X is an arbitrary space.

- Classifier: $c : X \rightarrow \{0,1\}$ = predictor

- Given $S = (X \times \{0,1\})^*$ find classifier c with small error rate

$$e(D,c) = \Pr_{(x,y)\sim D}(c(x) \neq y)$$

Note: D unknown. Impossible in general, but maybe possible in particular. We hope S drawn IID from D, but it isn't always so.

Importance Weighted Classification

- Problem: A measure D on $X \times \{0,1\} \times [0,\infty)$ where X is an arbitrary space.

- Classifier: $c : X \rightarrow \{0,1\}$ = predictor

- Given $S = (X \times \{0,1\}, [0,\infty))^*$ find classifier c with small importance weighted loss

$$e_w(D,c) = E_{(x,y,i)\sim D}[iI(c(x) \neq y)]$$

The core theorem: folklore

Theorem: (Distribution shift) For all c, for all importance weighted D, let $D'(x,y,i) = \frac{iD(x,y,i)}{E_{(x,y,i)\sim D}[i]}$. Then:

$$e_w(D,c) = e(D',c)E_{(x,y,i)\sim D}[i]$$

... so minimizing D error rate = minimizing D_i importance weighted error rate. Proof:

$e_w(D,c) = \sum_{(x,y,i)}[iD(x,y,i)I(c(x) \neq y)]$

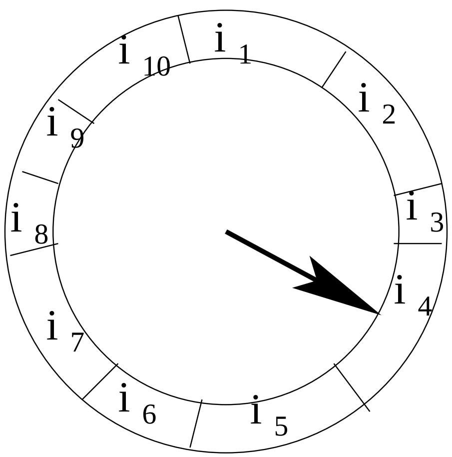
$= E_{(x,y,i)\sim D}[i] \sum_{(x,y,i)}[D'(x,y,i)I(c(x) \neq y)]$

$= E_{(x,y,i)\sim D}[i] \, \Pr_{(x,y)\sim D'}[I(c(x) \neq y)]$

$= E_{(x,y,i)\sim D}[i]e(D',c)$

How do we change distributions?

Something which doesn't work: Resampling



Resampling=place examples on roulette wheel with coverage proportional to weight. Spin the wheel many times.

Basic problem: duplicate examples = nonindependence.

Distribution Transform: Rejection Sampling.

1. Pick a constant c larger than any importance ($\forall i \;\; c > i$)

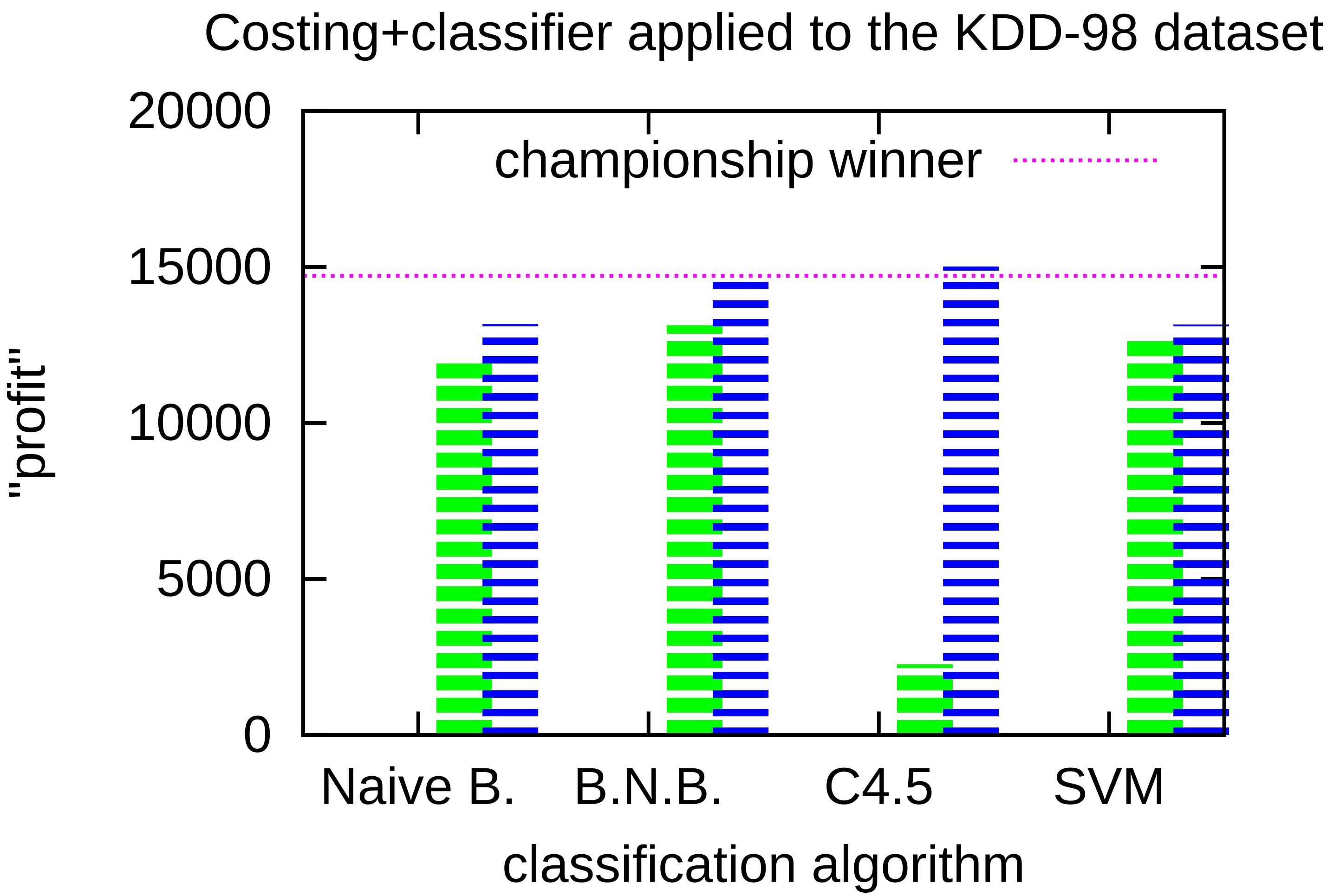
2. For each sample (x, y, i), flip a coin with bias $\frac{i}{c}$. If the result is "heads" keep it, and otherwise discard it.

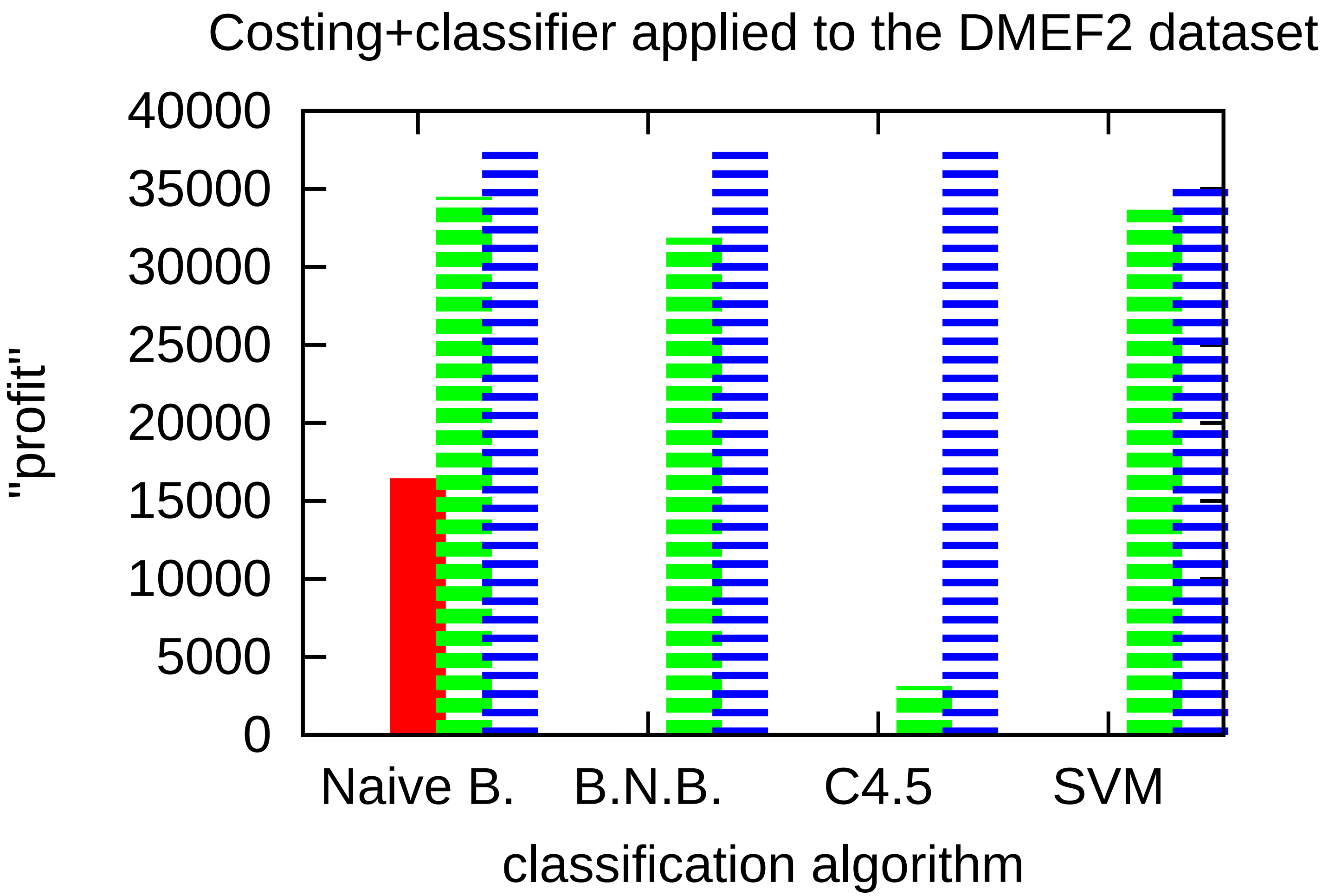
Rejection sampling $\Rightarrow$ samples in S are IID if samples in S_w are IID.

Costing(S_w,A)

1. For $t = 1$ to 10 do
   (a) Rejection sample to form S_t from S_w.
   (b) Learn $c_t = A(S_t)$

Output: $c(x) = \text{majority}(\{c_1(x), ..., c_{10}(x)\})$


Costing+classifier applied to the KDD-98 dataset
20000
15000
10000
5000
0
"profit"
championship winner
Naive B.
B.N.B.
C4.5
SVM
classification algorithm



Costing+classifier applied to the DMEF2 dataset
"profit"
40000
35000
30000
25000
20000
15000
10000
5000
0
Naive B.
B.N.B.
C4.5
SVM
classification algorithm

Given a Binary classifier, how can we solve

1. Importance Weighted Classification?

2. Class Probability estimation?

3. Multiclass Classification?

4. Cost Sensitive Classification?

5. Reinforcement Learning?

Class Probability Estimation

- Problem: A measure D on $X \times \{0, 1\}$ where X is an arbitrary space.

- Probabilistic Classifier: $c_p : X \to [0, 1]$ = predictor

- Given $S = (X \times \{0, 1\})^*$ find probabilistic classifier c_p with small squared error:

$$e_p(D, c_p) = E_{(x,y)\sim D}[(c_p(x) - y)^2]$$

Reasons for The Probability Estimation Problem

1. Doctor wants “advice” from a machine, but not a decision.

2. Distributed system requires efficient communication of beliefs.

3. Compatibility between prediction and probabilistic prediction worlds.

The Probing Method: Observations

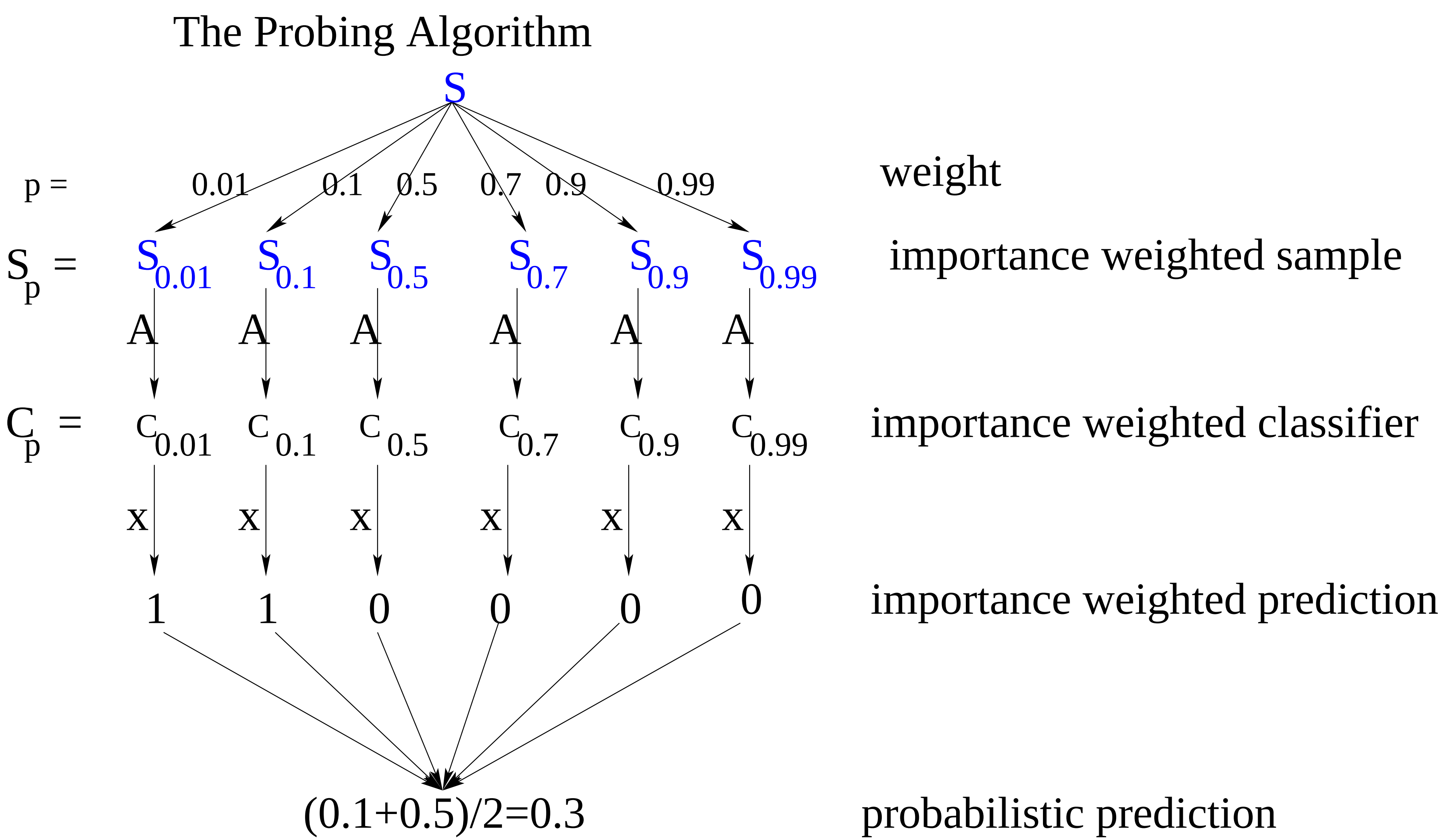
Observation: if c is perfect, $c(x) = 1 \Rightarrow D(y = 1|x) > 0.5$

1. Pick $p \in (0, 1)$.
2. Map $(x, y) \to (x, y, |y - p|)$ (= importance weighted example)

if c perfect then, $c(x) = 1 \Rightarrow D(y = 1|x) > p$

Proof:

Prediction	Expected Importance given x
0	$(1 - D(y = 1|x)p$
1	$D(y = 1|x)(1 - p)$



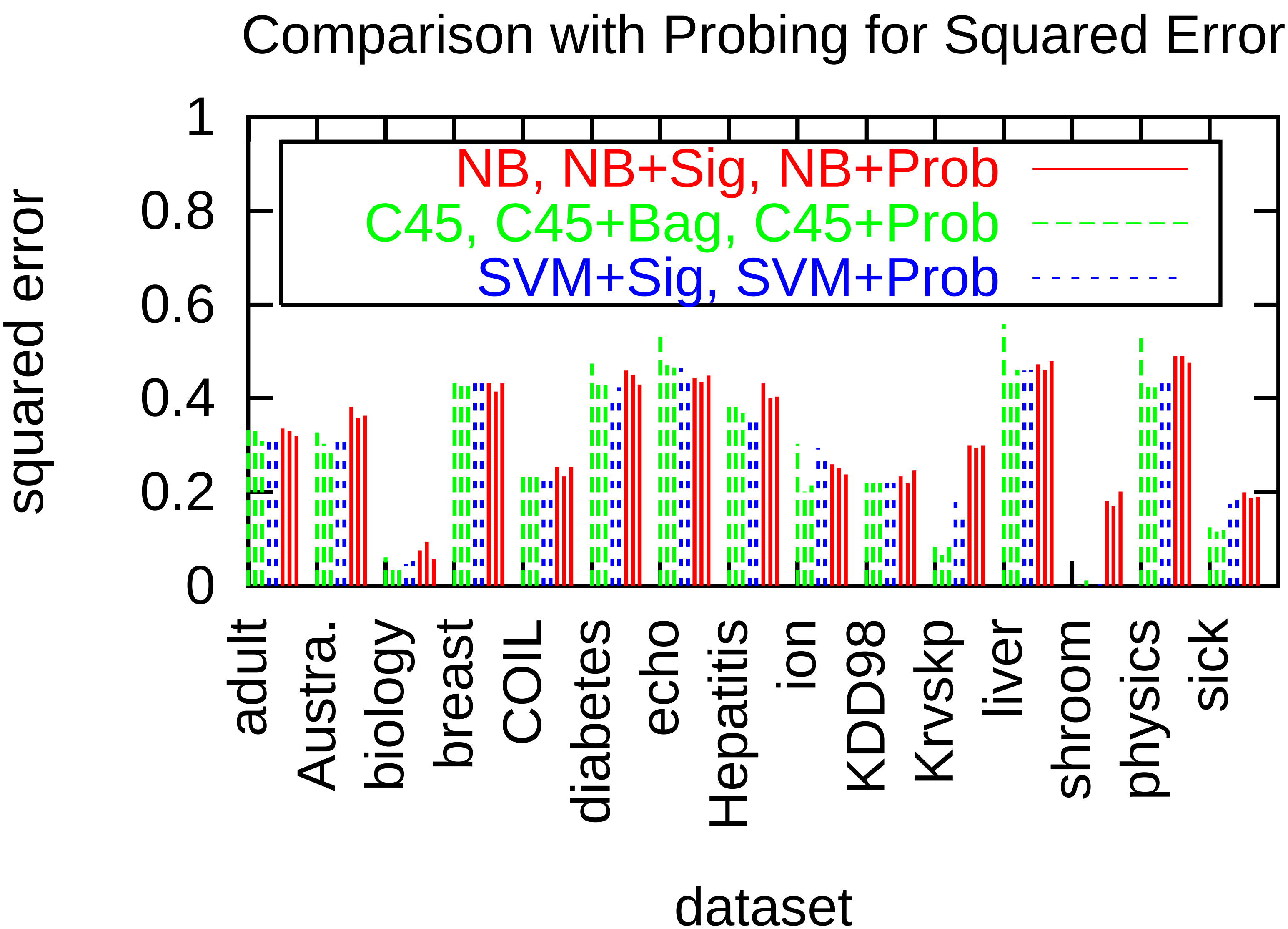
The Probing Algorithm
S
p =
0.01
0.1
0.5
0.7
0.9
0.99
weight
S_p =
S_0.01
S_0.1
S_0.5
S_0.7
S_0.9
S_0.99
importance weighted sample
A
C_p =
C_0.01
C_0.1
C_0.5
C_0.7
C_0.9
C_0.99
importance weighted classifier
x
1 1 0 0 0 0
importance weighted prediction
(0.1+0.5)/2=0.3
probabilistic prediction

The Probing Method: Details

1. How do you make classifier take weights? Costing Reduction

2. How do you deal with nonmonotonic predictions? Sort

3. How do you discretize on p? Uniform grid or on demand



Comparison with Probing for Squared Error
NB, NB+Sig, NB+Prob
C45, C45+Bag, C45+Prob
SVM+Sig, SVM+Prob
squared error
0
0.2
0.4
0.6
0.8
1
adult
Austra.
biology
breast
COIL
diabetes
echo
Hepatitis
ion
KDD98
Krvskp
liver
shroom
physics
sick
dataset

The one classifier trick

We learn many classifiers in parallel. They *could* be one classifier.

1. Let $S = \cup_p \{(< x, p >, y, i) : (x, y, i) \in S_p\}$

2. Let $c = \text{Costing}(S, A)$

3. Let $c_p(x) = c(x, p)$

Good for practice? Unknown.

Handy for theory: we can think about drawing from induced distribution on c by drawing from $(x, y) \sim D$ and $p \sim U(0, 1)$ to generate a sample from induced distribution $\text{Probing}(D)$.

Probing Theory

Cute observation: $\text{Probing}_c(x) = \text{Pr}_{p \sim U(0,1)}(c(x,p) = 1)$

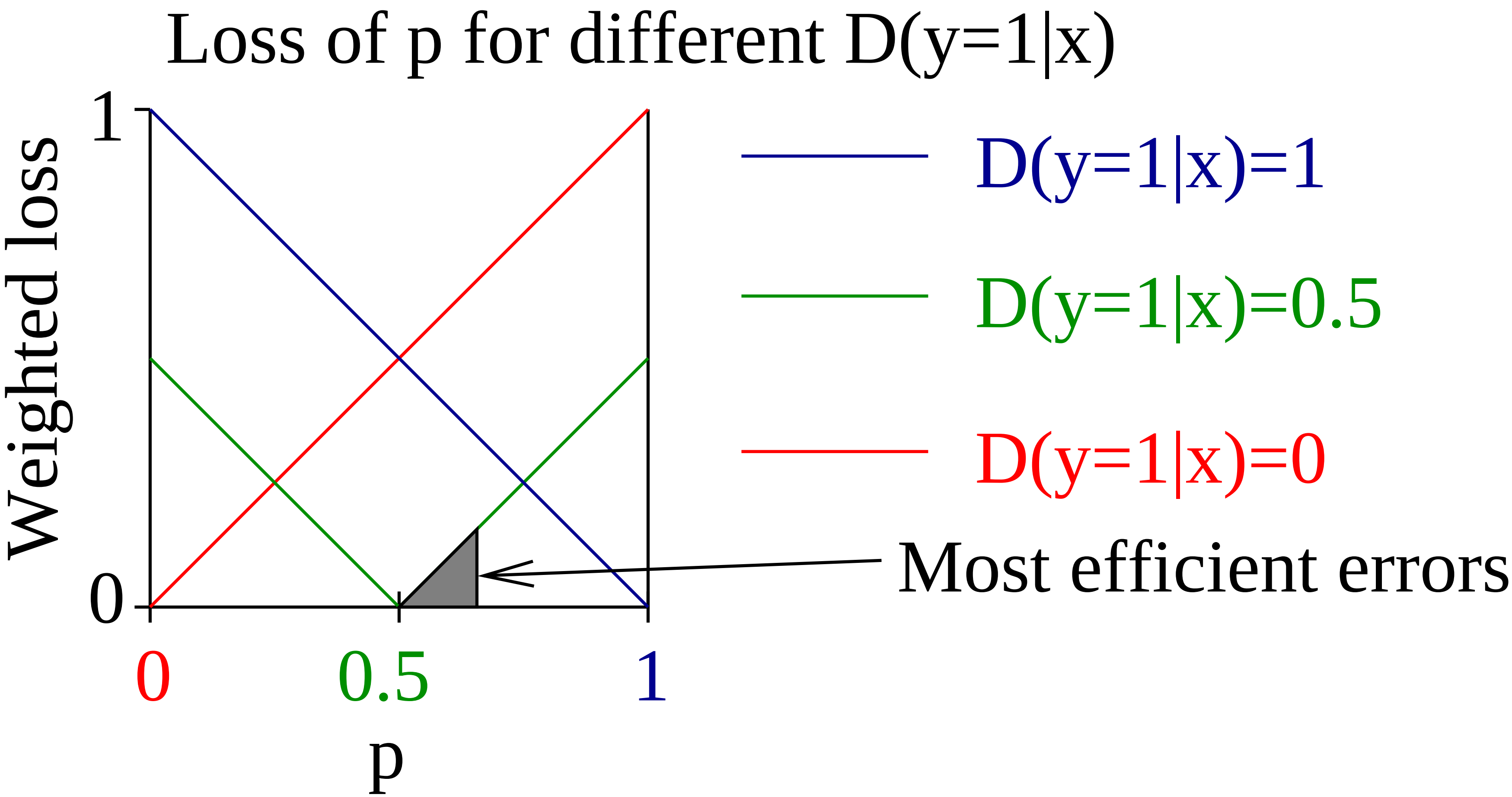
Theorem: (Probing Translation) For all $c : X \times [0,1] \to \{0,1\}$, for all D on $X \times \{0,1\}$

$$E_{x,y \sim D}(D(y|x) - \text{Probing}_c(x))^2$$

$$\leq e(\text{Probing}(D), c) - \min_{c'} e(\text{Probing}(D), c')$$

... spooky. You don't know $D(y|x)$, yet minimizing $e(\text{Probing}(D), c)$ always implies good estimates of $D(y|x)$.

The proof, pictorially



The proof, mathematically

Expected importance $= \frac{1}{2}$ so:

$e(\text{Probing}(D), c) - \min_{c'} e(\text{Probing}(D), c')$

$= 2E_{p,(x,y)\sim D}|y-p|I(c(x,p) \neq y) - \min\{(1-p)D(1|x), p(1-D(1|x))\}$

("2" comes from the distribution shift theorem)

$= 2E_{x,p}E_{y\sim D|x}|y-p|I(c(x,p) \neq y) - \min\{(1-p)D(1|x), p(1-D(1|x))\}$

For any x, p, either c predicts perfectly (difference $= 0$) or not.

If not, difference $= 2|(1-p)D(1|x) - p(1-D(1|x))| = 2|p - D(1|x)|$

$\Rightarrow$ cost of misclassification $= 2|p - D(1|x)|$.

Proof II: Properties of most efficient error inducing method

How can ϵ binary errors maximize probability estimation errors?

1. Only classifications $c(x,p)$ on one side of $D(1|x)$ err. (otherwise sort = cancellation of errors = wasted binary errors)

2. Errors for p closer to $D(1|x)$ are preferred over errors further from $D(1|x)$ (sort makes these equivalent, and the importance weighted loss is smaller for nearer errors)

$\Rightarrow$ deviation Δ requires at least importance weighted loss

$$\int_{D(1|x)}^{D(1|x)+\Delta} 2|D(1|x) - z|dz = \Delta^2$$

Some Caveats

1. No bound holds for cross entropy. (Can't be done without extra assumptions/constraints.)

2. No direct bound on error rate for relative ranking.

Given a Binary classifier, how can we solve

1. Importance Weighted Classification?

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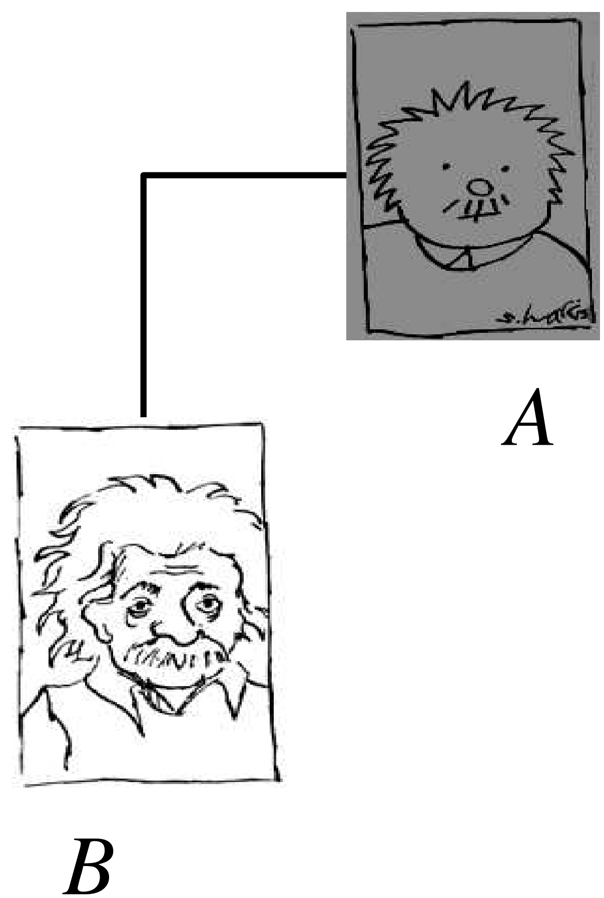
Multiclass Reductions

Reductions turn a (black-box) learner for *A* into a learner for *B*.



Online Learning

The number of mistakes we make is never more than f (number of mistakes made by the best expert so far)

Learning Reductions

If my oracle does well on its subproblems, I will do well on the original problem

PAC Learning

Concept class C, hypothesis class H.

The learner needs to observe at most poly many i.i.d. examples to output a hypothesis h in H that has small error with high probability.

Algorithms: consistency

VC Learning

Bayes Learning

Agnostic Learning

No assumptions
Weaker (relative) statements
Always applicable

Strong Assumptions
Strong Statements
Weak applicability

Multiclass Classification

- Problem: A measure D on $X \times \{1, ..., k\}$ where X is an arbitrary space.

- Multiclass Classifier: $c_m : X \to \{1, ..., k\}$ = predictor

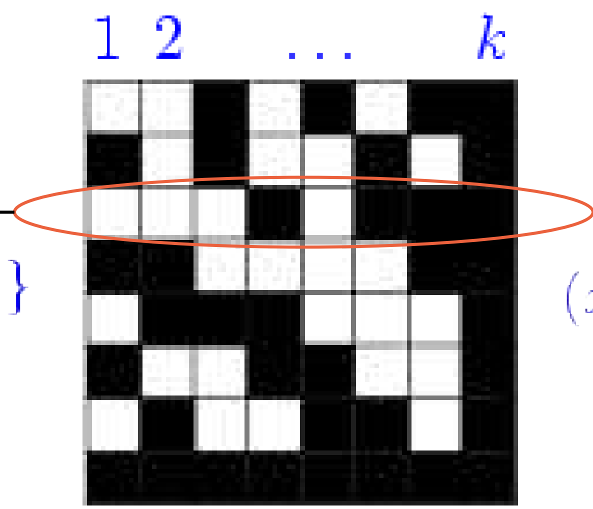
- Given $S = (X \times \{1, ..., k\})^*$ find multiclass classifier c_m with small error rate:

$$e(D, c_m) = \Pr_{(x,y)\sim D}(c_m(x) \neq y)$$

ECOC Transformation

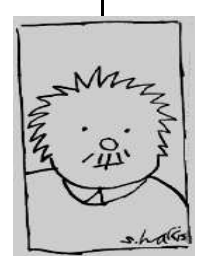
Multiclass examples

(x_i, y_i) $y_i \in \{1, \ldots, k\}$



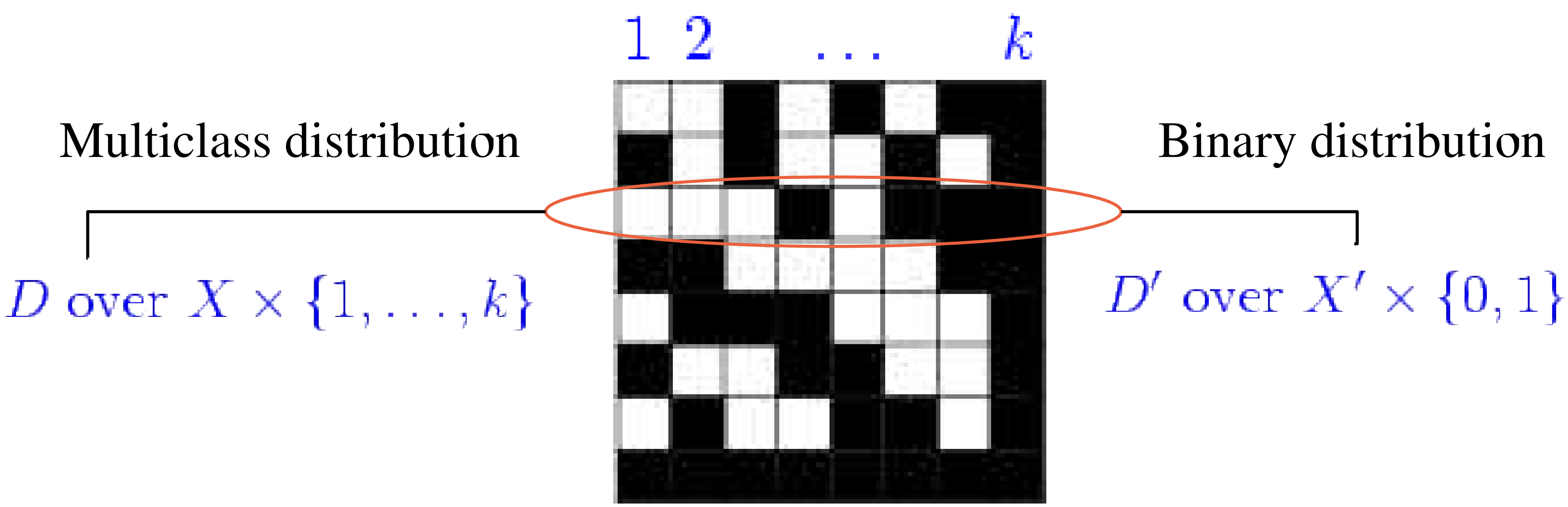
Binary examples

$(x_i, I(y_i \in \{1, 2, 3, 5\}))$



Each row (= subset of labels *S*)
defines a binary problem:
"Is the label in *S* or not?"

ECOC Transformation

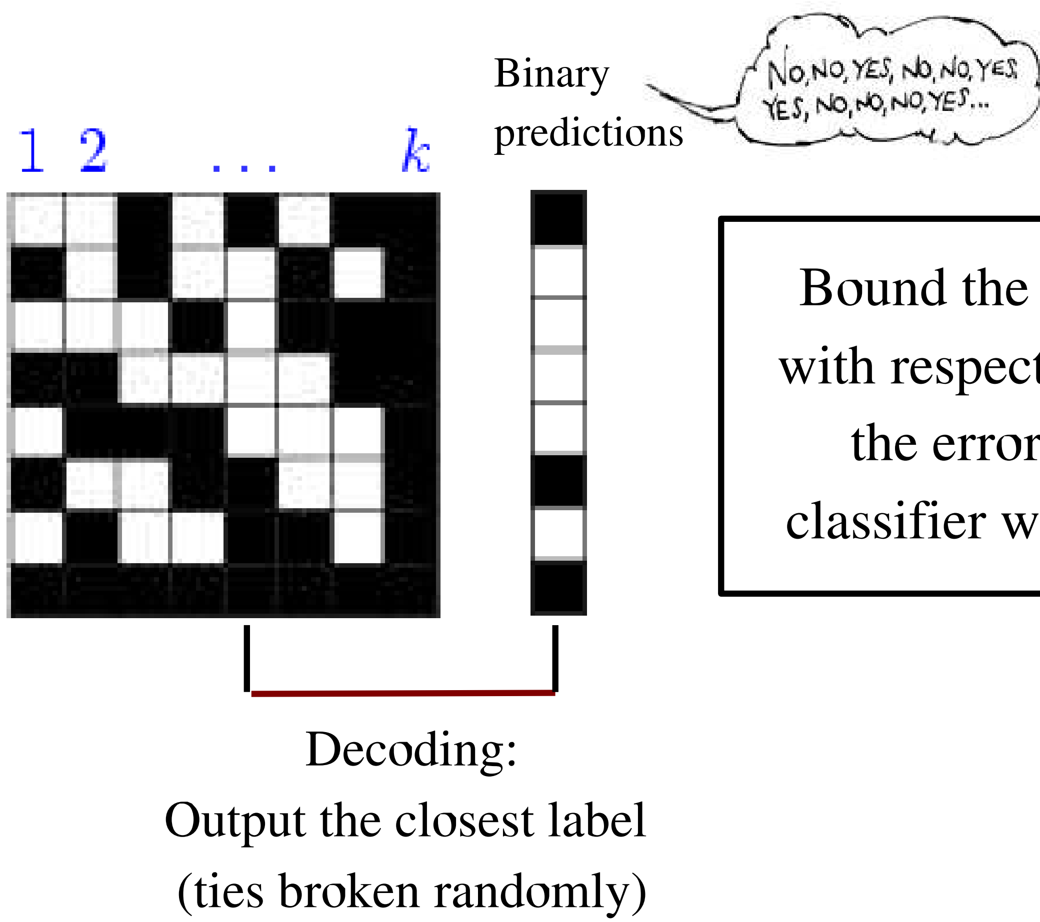


Draw $(x, y) \sim D$, a random row S, output $(\langle x, S \rangle, I(y \in S))$

Error Transformation

How does performance on D' imply performance on D ?

ECOC Prediction



Bound the multiclass error with respect to D in terms of the error of the binary classifier with respect to D'.

ECOC Analysis

Use one classifier trick $c(x, s) = c_s(x)$. Let $\text{ECOC}(D_m)$= induced distribution on binary predictions.

Theorem: (ECOC transform) For all k, there exists code such that for all c, for all D_m on (x, y_m),

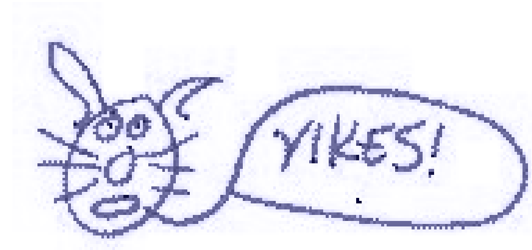
$$e(D_m, \text{ECOC}_c) \leq 4e(\text{ECOC}(D_m), c)$$

Proof: Exists code such that $< \frac{1}{4}$ binary errs $\Rightarrow$ no error

Intuition: two random bit vectors disagree in $\frac{1}{2}$ of locations so $\frac{1}{4}$ positions must be flipped to change which vector is nearest.

Exact example = rows of Hadamard matrix.

Two little problems



1) Not interesting if the error rate of the oracle is > ¼ .

2) Inconsistency:

ECOC Inconsistency: There exists *D* such that for all codes,

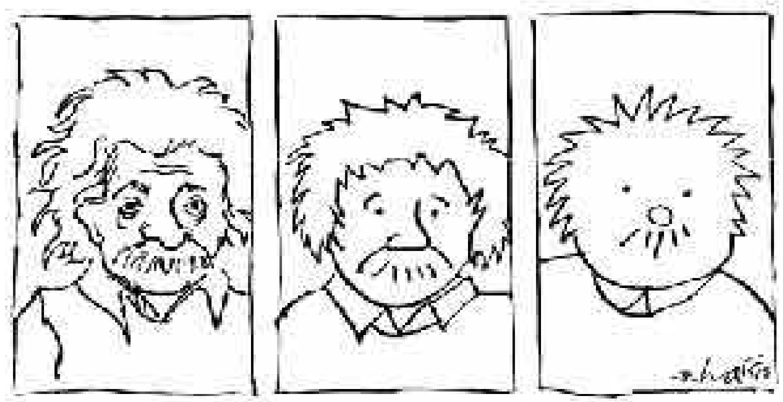
ECOC with $c^* = \text{argmin}_c\, e(\text{ECOC}(D), c)$

fails to provide an optimal multiclass classifier:

$$e(D, \text{ECOC}_{c^*}) - \min_h e(D, h) > 0$$

PECOC: Probabilistic ECOC

multiclass



binary

binary probability estimation

Probabilistic Error Correcting Output Code

As ECOC, except use Probing to get probabilistic predictions.

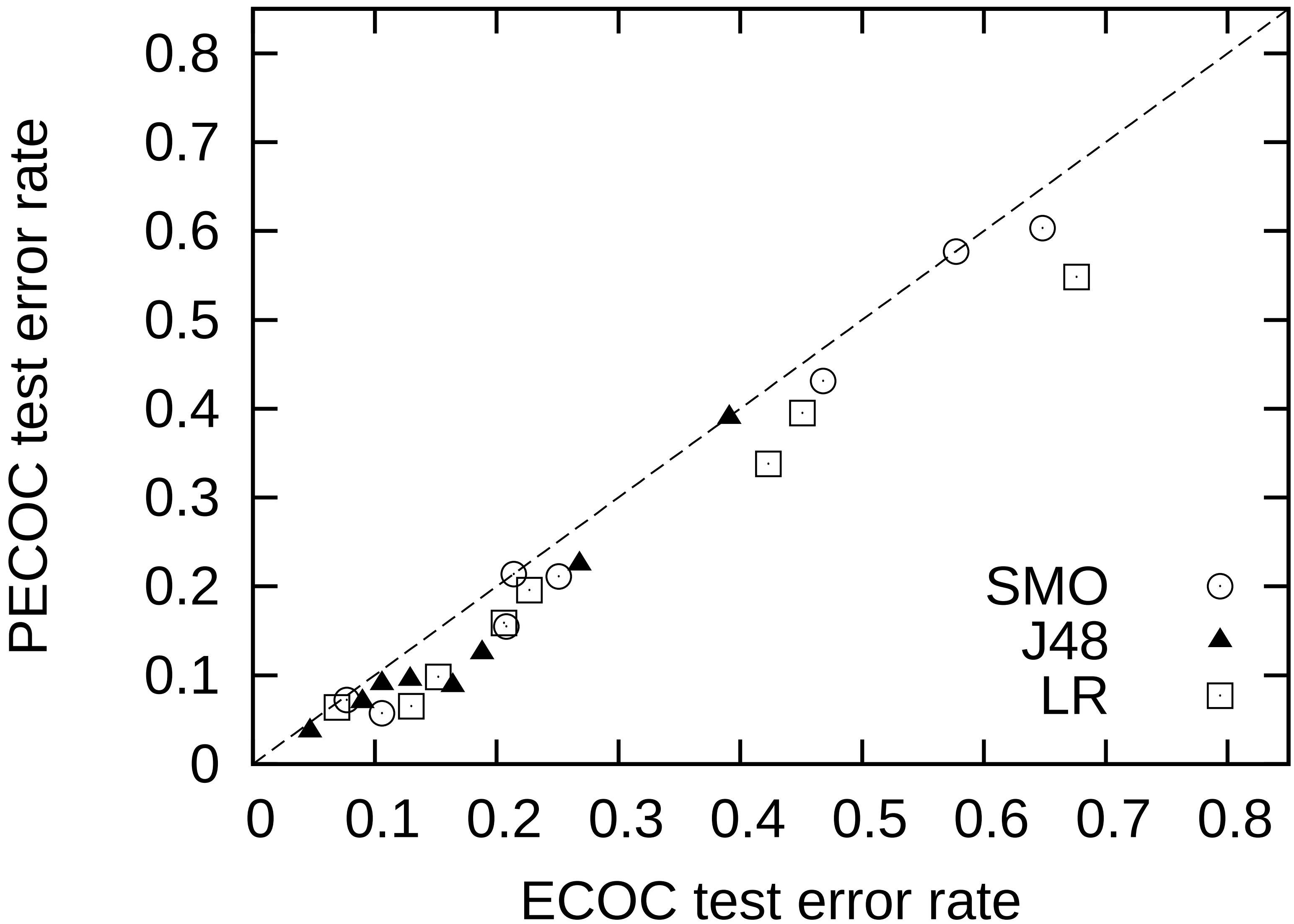
To predict: Use l_1 closest label/codeword.

Label

Binary Problem	1	2	3	4
	1	1	0	0
	1	0	1	0
	1	0	0	1

predictions
0.91
0.55
0.46

	1	2	3	4
	0.09	0.09	0.91	0.91
	0.45	0.55	0.45	0.55
	0.54	0.46	0.46	0.54
sum	1.08	1.10	1.82	2.00


PECOC test error rate
ECOC test error rate
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
SMO
J48
LR

PECOC Analysis

Use one classifier trick $c(x,s,p) = c_{sp}(x)$. Let $\text{PECOC}(D_m)$= induced distribution on binary predictions.

Theorem: (PECOC transform) For all k, there exists code such that for all c, for all D_m on (x, y_m),

$$e(D_m, \text{PECOC}_c) - \min_{c'} e(D_m, c')$$

$$\leq 4\sqrt{e(\text{PECOC}(D_m), c) - \min_{c'} e(\text{PECOC}(D_m), c')}$$

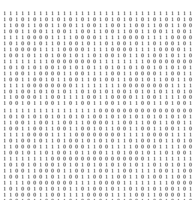
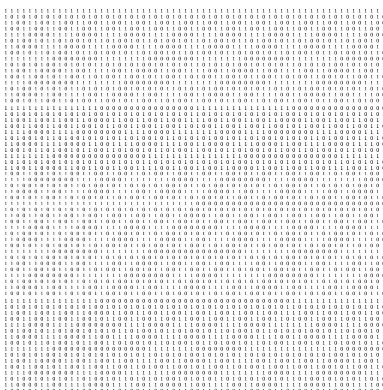
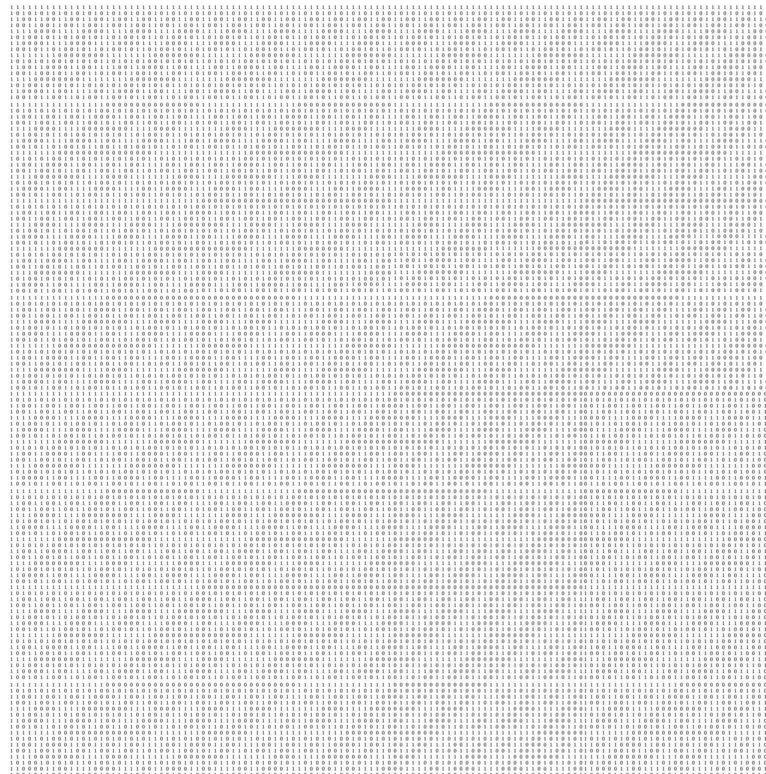
- Binary consistency $\Rightarrow$ Multiclass consistency

- Binary error rate $= 0.25$ OK when minimum error rate $= 0.25$

Proof

Pick code = subset of recursively defined Hadamard matrix.

1) $\begin{matrix} 1 & 1 \\ 1 & 0 \end{matrix}$
2) $\begin{matrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{matrix}$
3) $\begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{matrix}$
4) $\begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{matrix}$

5)  6)  7)  8)

...

Hadamard was a rug designer!

If b = number of binary problems, distance between codewords $= \frac{b}{2}$

For all labels l,

$$\sum_{b \in \text{Binary Problems}} \text{Probing}_b(\text{set containing } l|x)$$

$$= \sum_{b \in \text{Binary Problems}} \left[\sum_{l' \in \text{set containing l}} D_m(l'|x) \right]$$

(Assuming perfect classifiers)

$$= b(D_m(l|x) + \frac{1}{2} \sum_{l' \neq l} D_m(l'|x)) = b\frac{D_m(l|x) + 1}{2}$$

Proof: Analyzing errors

From Probing analysis,

$$E_{x,y\sim D}|D(1|x) - \text{Probing}_c(x)|$$

$$\leq \sqrt{e(\text{Probing}(D), c) - \min_{c'} e(\text{Probing}(D), c')}$$

Let b = number of binary problems.

Most efficient method to disturb l_1 sum by ϵb is for each call to probing to err by $|D(1|x) - \text{Probing}(x)| = \epsilon$. (since $\sqrt{x}$ is convex)

Changing l_1 sum by $b\epsilon \Rightarrow$ choosing class with error rate at most 4ϵ worse than minimum.

Bonus!

You can also do probabilistic multiclass prediction.

$$\sum_{b \in \text{Binary Problems}} \hat{p}_b = b\frac{D_m(l|x) + 1}{2}$$

$$\Rightarrow D_m(l|x) = \frac{2\sum_{b \in \text{Binary Problems}} \hat{p}_b}{b} - 1$$

... and this turns out to be stable.

Theorem: (PECOC' Translation) For all $c : X \times [0, 1] \to \{0, 1\}$, for all D_m on $X \times \{1, ..., k\}$

$$E_{x,y \sim D}(D_m(y|x) - \text{PECOC'}_c(x))^2$$

$$\leq 4(e(\text{PECOC}(D_m), c) - \min_{c'} e(\text{PECOC}(D_m), c'))$$

Given a Binary classifier, how can we solve

1. Importance Weighted Classification?
2. Class Probability estimation?
3. Multiclass Classification?
4. Cost Sensitive Classification?
5. Reinforcement Learning?

Cost Sensitive Classification

- Problem: A measure D_{cs} on $X \times [0, \infty)^k$ where X is an arbitrary space.

- Multiclass Classifier: $c_m : X \to \{1, ..., k\}$ = predictor

- Given $S = (X \times [0, \infty)^k)^*$ find classifier with small expected loss

$$e_{cs}(D_{cs}, c_m) = E_{(x,\vec{\ell}) \sim D_{cs}}[\ell_{c_m(x)}]$$

Sensitive Error Correcting Output Code

SECOC = cost sensitive ⇒ importance weighted classification reduction, in two parts. Uses a code matrix M:

Subset \ Label	1	2	3	4
	1	1	0	0
	1	0	1	0
	1	0	0	1

SECOC, the training algorithm

SECOC-Train (cost sensitive examples S, importance weighted learning algorithm A)

1. For each subset s defined by the rows of M:
   (a) For $(x, \vec{\ell}) \in S$, let $|\vec{\ell}| = \sum_y \ell_y$ and $\ell_s = \sum_{y \in s} \ell_y$.
   (b) For random t in $[0, 1]$:
   Let $c_{st} = A(\{(x,\ I(\ell_s \geq t|\vec{\ell}|), |\ell_s - |\vec{\ell}|t|) : (x,\ \vec{\ell}) \in S\})$.
2. return $\{\ell_{st}\}$

SECOC, the prediction algorithm

SECOC-Predict (classifiers $\{c_{st}\}$, example $x \in X$)

return $\min_y E_s E_t \left[I(y \in s) c_{st}(x) + I(y \notin s)(1 - c_{st}(x)) \right]$

SECOC Analysis

Use the one classifier trick + Costing. Let

$$S'_{st} = \{((x,s,t),y,i) : (x,y,i) \in S_{st}\}$$

Then train to learn $c = A(\cup_{st} S'_{st})$ and define $c_{st}(x) = c(x,s,t)$

Theorem: (SECOC transform) For all k, there exists code such that for all c, for all D_{cs} on $(x, \vec{\ell})$,

$$e_{cs}(D_{cs}, \text{SECOC}_c) - \min_{c'} e_{cs}(D_{cs}, c')$$

$$\leq 4\sqrt{(e(\text{SECOC}(D_{cs}), c) - \min_{c'} e(\text{SECOC}(D_{cs}), c'))E_{(x,\vec{\ell})\sim D_{cs}}|\vec{\ell}|}$$

Proof (sketch only, much like PECOC)

1. $E_t\left[I(y \in s)c_{st}(x) + I(y \notin s)(1 - c_{st}(x))\right] = \frac{E_{\vec{\ell}\sim D|x}[\ell_s]}{E_{\vec{\ell}\sim D|x}\left[|\vec{\ell}|\right]}$ when classifiers optimal.

2. $E_s \frac{E_{\vec{\ell}\sim D|x}[\ell_s]}{E_{\vec{\ell}\sim D|x}\left[|\vec{\ell}|\right]} = \frac{\frac{E_{\vec{\ell}\sim D|x}[\ell_y]}{E_{\vec{\ell}\sim D|x}\left[|\vec{\ell}|\right]}+1}{2}$ when classifiers optimal.

3. Optimal method for adversary to cause loss without incurring importance weighted regret = small error t's for each s.

4. Cost of erring linear in $t \Rightarrow$ average regret growth quadratic.

Another Bonus

Corollary: Soft Prediction

$$\Rightarrow E_{x\sim D_{cs}}\left(\text{Predict}(c,x,y)E_{\vec{\ell'}\sim D|x}\left[|\vec{\ell'}|\right] - E_{\vec{\ell'}\sim D|x}\left[\ell'_y\right]\right)^2$$

$$\leq 4(e(\text{SECOC}(D_{cs}),c) - \min_{c'} e(\text{SECOC}(D_{cs}),c'))E_{(x,\vec{\ell})\sim D_{cs}}|\vec{\ell}|$$

Some Things to Think About

Experiments: Not done yet. How well does this work in practice?

Which code do you use in practice?

Shannon $\Rightarrow$ a random code of size $O(\log k)$ is near optimal for above theory.

Given a Binary classifier, how can we solve

1. Importance Weighted Classification?
2. Class Probability estimation?
3. Multiclass Classification?
4. Cost Sensitive Classification?
5. Reinforcement Learning?

Reinforcement Learning $\Rightarrow$ classification status

1. We aren't there yet.

2. We are there, *given* access to a "generative model".

3. We are there, *given* oracle access to the optimal policy.

Reinforcement Learning Problem

$D(o', r|(o, a, r)^*)$ = conditional probability table

1. $o', o \in O$ = observations

2. $r \in [0, \infty)$ = reward

3. $a \in A$ = action:

Find $\pi((o, a, r)^*, o) \to a$ maximizing:

$$\eta(\pi) = E_{(o,a,r)^T \sim \pi, D} \sum_{t=1}^{T} r_t$$

“Algorithm”

Let π^*= optimal policy

1. Given empty history, predict optimal a assuming π^* followed for $T-1$ timesteps afterwards.

2. Given first prediction, 1 step history, predict optimal a assuming π^* followed for $T-2$ timesteps afterwards.

3. Given 2 step history, predict optimal a assuming π^* followed for $T-3$ timesteps afterwards.

4. ...

Analysis

$\rho(\pi) = \eta(\pi^*) - \eta(\pi)$ = policy regret

$l_t(D, \pi, c_t) = E_{(a,o,r)^T \; D,(\pi, c_t, \pi^*)} \sum_{t'=t}^{T} r_{t'}$

$\rho_t(D, \pi, c_t) = l_t(D, \pi, \pi_t^*) - l_t(D, \pi, c_t)$

Theorem: For all D, $\pi = (c_1, ..., c_T)$

$$\rho(D, \pi) \leq 4 \sum_{t=1}^{T} \sqrt{\text{binary regret}(\text{SECOC}(D_t), c_t) \sum_a \rho_t(D_t, \pi, c_a)}$$

Significance: Quantification of RL performance in terms of classification performance.

Proof sketch

Definition manipulation + telescoping sum

$$\Rightarrow \rho(D, \pi) = \sum_{t=1}^{T} \rho(D_t, c_t)$$

Then note that each choice is a cost sensitive classification problem and apply SECOC theorem.

A Use For Greg Grudic

Teleoperated robot $\Rightarrow$ human = near optimal policy...

Algorithm:

1. Robot acts.
2. Human completes.
3. Reset robot and go to (1).

Observe rewards, compute samples, apply classification.

Final Thoughts

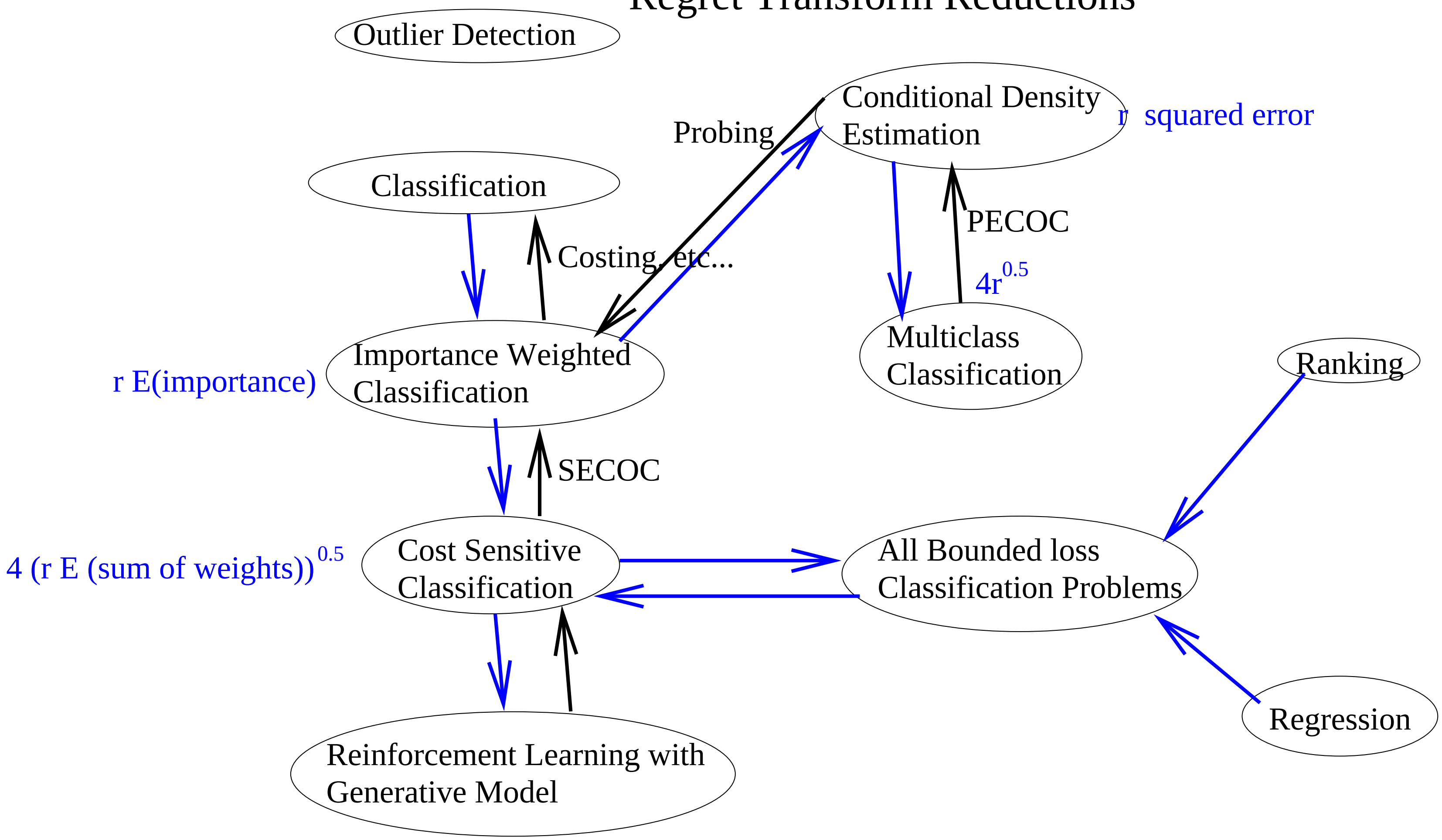
Formal study of learning reductions is relatively new.

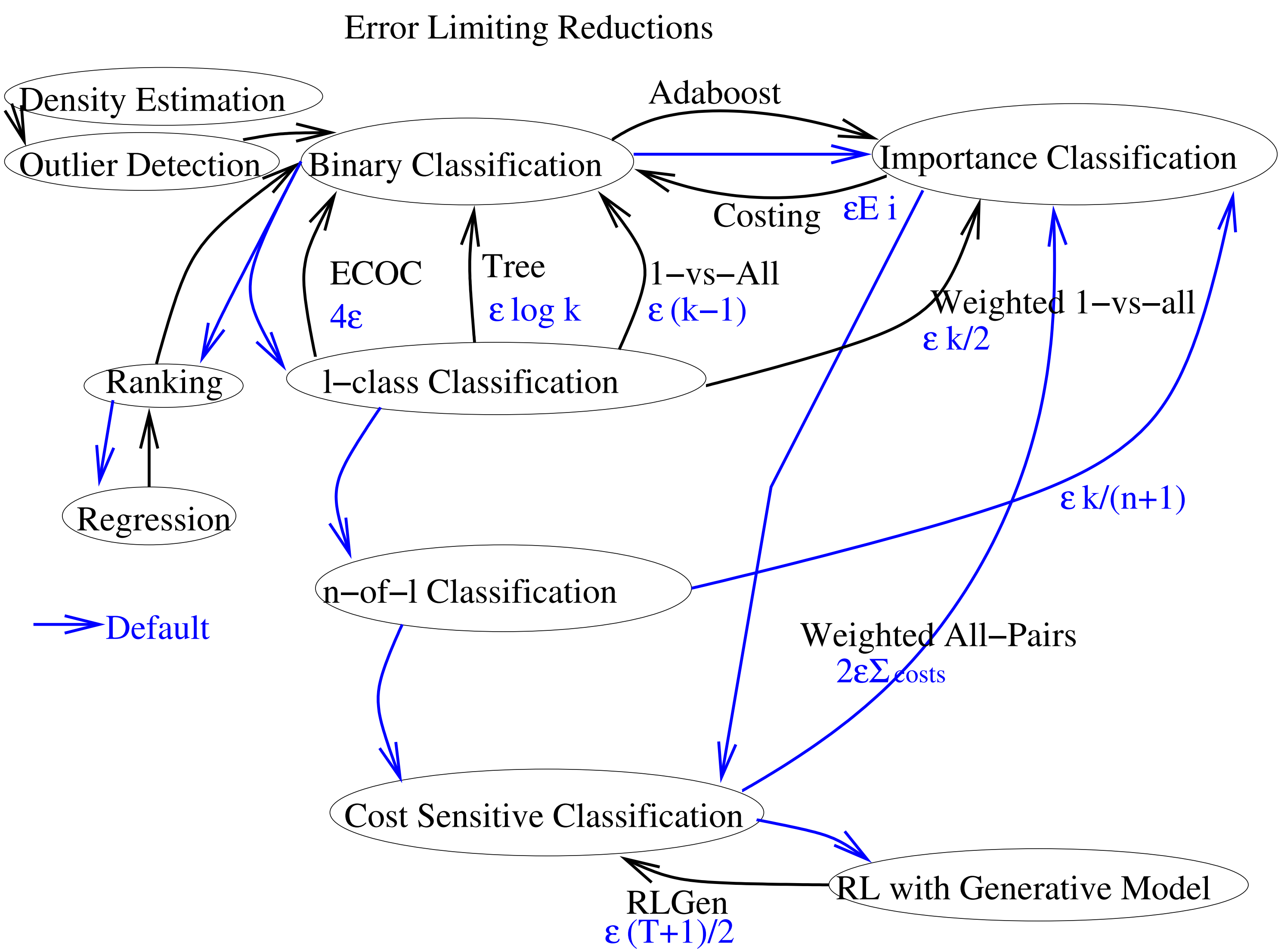
- $\Rightarrow$ The limits of the possible are not entirely known.
- $\Rightarrow$ You-a-fellow-researcher could easily have important insights.

Coherent codebase, coming soon.

Coherent tutorial in paper form, coming soon.

Regret Transform Reductions




Error Limiting Reductions
Density Estimation
Outlier Detection
Binary Classification
Importance Classification
Adaboost
Costing
εE i
ECOC
4ε
Tree
ε log k
1−vs−All
ε (k−1)
Weighted 1−vs−all
ε k/2
Ranking
l−class Classification
Regression
ε k/(n+1)
n−of−l Classification
Default
Weighted All−Pairs
2εΣcosts
Cost Sensitive Classification
RL with Generative Model
RLGen
ε (T+1)/2

Related Reading

[ECOC] T. Dietterich and G. Bakiri, "Solving Multiclass Classification via Error-Correcting Output Codes", JAIR 2:263-296, 1995.

[Boosting] Y. Freund and R. Schapire, "A Decision-Theoretic Generalization of Online Learning and an Application to Boosting", JCSS 55(1) 119-39, 1997.

[ECOC analysis] V. Guruswami and A. Sahai, "Multiclass Learning, boosting, and error-correcting codes", COLT 145-55, 1999.

[ECOC variant] E. Allwein, R. Schapire, and Y. Singer, "Reducing Multiclass to Binary: A Unifying Approach for Margin Classifiers", JMLR, 1:113-41, 2000.

[Costing] B. Zadrozny, J. Langford, and N. Abe. Cost-Sensitive L earning by Cost-Proportionate Example Weighting. ICDM 435–42, 2003.

[Probing] J. Langford and B. Zadrozny, “Estimating Class Membership Probabilities Using Classifier Learners”, AISTAT 2005.

[PECOC & SECOC] J. Langford and A. Beygelzimer, “Sensitive Error Correcting Output Codes”, COLT 2005.

[RL→Binary] J. Langford and B. Zadrozny, “Relating Reinforcement Learning Performance to Classification Performance”, ICML 2005.

See: http://hunch.net/~jl/projects/reductions/reductions.html


THANK YOU FOR NOT DOING RESEARCH THAT HAS ALREADY BEEN DONE
S. Harris