A TIGHT TRANSDUCTION TO INDUCTION REDUCTION

The PAC-MDL bound on the test error rate of a chosen classifier is:

$$\Pr_{S_{\text{test}},S_{\text{train}}} \left(\hat{c}_{\text{test}} \leq \overline{\text{Hyp}}(\hat{c}_{\text{train}}, |S_{\text{test}}|, \delta)\right) \geq 1-\delta$$

We also know that for every classifier:

$$\Pr_{S_{\text{test}},S_{\text{train}}} (\hat{c}_{\text{test}} \leq \text{Bin}(c_D, |S_{\text{test}}|, 1-\delta)) \geq 1-\delta$$

Next we state a theorem for one classifier. To get a theorem which holds for all classifiers, we can take a max over transductive classifiers consistent with the actual classifier.

Theorem 0.1.

$$\Pr_{S_{train}} \left(c_D \leq \overline{Bin}\left(\overline{Hyp}\left(\hat{c}_{train}, |S_{test}|, \frac{\delta}{2}\right), \frac{1}{2}\right)\right) \geq 1-\delta$$

Proof.

$$E_{X,Y} I(\Phi(X,Y)) > 1-\epsilon$$

□

$$\Rightarrow \forall \alpha > 1: \quad E_X I(E_Y I(\Phi(X,Y)) > 1-\alpha\epsilon) > 1-\frac{1}{\alpha}$$

so:

$$\Pr_{S_{\text{test}},S_{\text{train}}} \left(\hat{c}_{\text{test}} \leq \overline{\text{Hyp}}(\hat{c}_{\text{train}}, |S_{\text{test}}|, \frac{\delta}{2})\right) \geq 1-\frac{\delta}{2}$$

implies:

$$\Pr_{S_{\text{train}}} \left(\Pr_{S_{\text{test}}} \left(\hat{c}_{\text{test}} \leq \overline{\text{Hyp}}(\hat{c}_{\text{train}}, |S_{\text{test}}|, \frac{\delta}{2})\right) > \frac{1}{2}\right) \geq 1-\delta$$

$$\Pr_{S_{\text{train}}} \left(\text{Bin}(c_D, \frac{1}{2}) \geq \overline{\text{Hyp}}(\hat{c}_{\text{train}}, |S_{\text{test}}|, \frac{\delta}{2})\right) \geq 1-\delta$$

$$\Pr_{S_{\text{train}}} \left(c_{\text{D}} \leq \overline{\text{Bin}}(\overline{\text{Hyp}}(\hat{c}_{\text{train}}, |S_{\text{test}}|, \frac{\delta}{2}), \frac{1}{2})\right) \geq 1-\delta$$