$$\bar{F}(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\frac{1}{x}$$

for large x. Finding the inverse of $\bar{F}$ is difficult, but fortunately we don't have to as we are only interested in a bound. Note that:

$$\hat{\bar{F}}(x) \equiv \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} > \bar{F}(x)$$

which implies that

$$\hat{\bar{F}}^{-1}(x) > \bar{F}^{-1}(x)$$

$$\hat{\bar{F}}^{-1}(\epsilon) = \sqrt{2\ln\frac{1}{\epsilon} - \ln 2\pi}$$

Consequently,

$$O\left(\ln\frac{1}{\bar{F}\left(\frac{\bar{F}^{-1}(\epsilon)}{\gamma}\right)} + \ln\frac{m+1}{\delta}\right) \leq O\left(\ln\frac{1}{\bar{F}\left(\frac{\hat{\bar{F}}^{-1}(\epsilon)}{\gamma}\right)} + \ln\frac{m+1}{\delta}\right)$$

$$= O\left(\ln\frac{1}{\bar{F}\left(\frac{\sqrt{2\ln\frac{1}{\epsilon} - \ln 2\pi}}{\gamma}\right)} + \ln\frac{m}{\delta}\right)$$

$$= O\left(\ln\frac{\sqrt{2\pi}\sqrt{2\ln\frac{1}{\epsilon} - \ln 2\pi}}{\gamma e^{-\frac{2\ln\frac{1}{\epsilon} - \ln 2\pi}{2\gamma^2}}} + \ln\frac{m}{\delta}\right)$$

$$= O\left(\ln e^{\frac{2\ln\frac{1}{\epsilon} - \ln 2\pi}{2\gamma^2}} + \ln\ln\frac{1}{\epsilon} + \ln\frac{1}{\gamma} + \ln\frac{m}{\delta}\right)$$

$$= O\left(\frac{\ln\frac{1}{\epsilon}}{\gamma^2} + \ln\ln\frac{1}{\epsilon} + \ln\frac{1}{\gamma} + \ln\frac{m}{\delta}\right)$$

$$= O\left(\frac{\ln\frac{1}{\epsilon}}{\gamma^2} + \ln\frac{1}{\gamma} + \ln\frac{m}{\delta}\right)$$

$$= O\left(\frac{\ln\frac{1}{\epsilon}}{\gamma^2} + \ln\frac{1}{\gamma} + \ln\frac{m}{\delta}\right)$$

Using the Hoeffding approximation for the tail inequality, we get:

$$\Rightarrow e_Q \leq \hat{e}_\gamma + O\left(\lim_{\epsilon\to 0}\left(\frac{\frac{1}{\gamma^2}\ln\frac{1}{\epsilon} + \ln\frac{1}{\gamma} + \ln\frac{m}{\delta}}{m}\right)^{\frac{1}{2}}\right)$$