COMS-4771 Spring 2008

**Paper 1 Quiz** Name: ______________________________

**Kalai and Vempala**, *Efficient algorithms for the online decision problem* (2003).

You have 30 minutes to complete the questions. The quiz is worth 10 points.

**Question 1** (5 points): Construct an example (i.e., a cost sequence) showing that the FPL algorithm breaks down when we can only approximately solve the offline optimization problem efficiently.

Let the decision set be $S = \{1, \dots, n\}$. Consider the cost sequence $(1, 1, \dots, 1)$ repeated $T/(n+1)$ times, $(1, 0, \dots, 0)$ repeated $T/(n+1)$ times, ..., $(0, \dots, 0, 1)$ repeated $T/(n+1)$ times. Now consider the algorithm that selects a decision of cost 1 at each step. This algorithm is always a 2-approximation to the leader on the previous examples. The cost of this algorithm is $T$ while the cost of the best $s \in S$ is $2T/(n+1)$, giving large regret. Adding perturbations of $O(\sqrt{T})$ as in the FPL will not significantly improve the regret (while the algorithm is still, say, a 3-approximation for sufficiently large $T$).

**Question 2** (5 points): Consider the online path-selection problem in an undirected graph where the edges have binary costs (they either fail or they don't), and the cost of a path is 1 if any edge on the path fails, 0 otherwise.

- Is there an efficient algorithm for computing the best static offline solution?

  Answer: The cost function here is non-linear; computing the best static offline solution NP-hard (and also hard to approximate).

- What does the paper imply about the setting where the costs of all edges are revealed at the end of each time step?

  There is no efficient offline algorithm (even if there was an efficient approximation, the results would break down). If we don't insist that the oracle is efficient, we can view each path as an expert. The number of such experts is exponential in the number of nodes $n$, and the additive mistake in this case is $O(\sqrt{n/T})$.

- What about the setting where the only feedback is the cost of the chosen path?

  Nothing.