Active Learning

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Machine Learning the Future Class, May 1
An instrument of mass machine learning

How can we formalize it’s use?
A lot of unlabeled data is plentiful and cheap, eg.

documents off the web

speech samples

images and video

*But labeling can be expensive.*
Exploiting unlabeled data

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Unlabeled points
Exploiting unlabeled data

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*But labeling can be expensive.*
Can interaction help us learn effectively?

The Active Learning Setting

Repeatedly:

1. Observe unlabeled example $x$.
2. Asking for label? Yes/no
3. If yes, observe label $y$.

Goal: Simultaneously optimize quality of learned classifier and minimize the number of labels requested.
Typical heuristics for active learning

Start with a pool of unlabeled data
Pick a few points at random and get their labels
Repeat
  Fit a classifier to the labels seen so far
  Query the unlabeled point that is closest to the boundary
  (or most uncertain, or most likely to decrease overall uncertainty,...)
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Biased sampling: the labeled points are not representative of the underlying distribution!
Sampling bias

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Query the unlabeled point that is closest to the boundary
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Example:

```
| 45% | 5% | 5% | 45% |
```

Even with infinitely many labels, converges to a classifier with 5% error instead of the best achievable, 2.5%.

Not consistent!
This problem occurs in practice.
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This problem occurs in practice.
Outline

1. Importance weighting
2. Rare Classes
3. Cost Sensitive
While (unlabeled examples remain)

1. Receive unlabeled example $x$.
2. Choose a probability of labeling $p$.
3. With probability $p$ get label $y$, and add $(x, y, \frac{1}{p})$ to $S$.
4. Let $h = \text{Learn}(S)$.

Consistency Theorem: For all methods choosing $p > 0$, the algorithm is consistent.
How should $p$ be chosen?

On the $t$th unlabeled point

let: $\hat{e}(h, S) = \frac{1}{t} \sum_{(x, y, i) \in S} \mathbb{1}(h(x) \neq y) = \text{importance weighted error rate.}$
How should \( p \) be chosen?

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let: \( \hat{e}(h, S) = \frac{1}{t} \sum_{(x,y,i) \in S} \mathbb{1}(h(x) \neq y) \) = importance weighted error rate.
Let \( h' = \) minimum error rate hypothesis choosing other label.
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let: $\hat{e}(h, S) = \frac{1}{t} \sum_{(x,y,i) \in S} i(1)(h(x) \neq y)$ = importance weighted error rate.

Let $h' = \text{minimum error rate hypothesis choosing other label}.$

Let $\Delta = \hat{e}(h', S) - \hat{e}(h, S)$ = error rate difference.
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Choose $p = 1$ if $\Delta \leq O\left(\sqrt{\frac{\log t}{t}}\right)$

Otherwise, let $p = O\left(\frac{\log t}{\Delta^2 t}\right)$
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**On the \( t \)th unlabeled point**

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**Accuracy Theorem:** With high probability, the IWAL reduction has a similar error rate to supervised learning on \( t \) points.
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Accuracy Theorem: With high probability, the IWAL reduction has a similar error rate to supervised learning on $t$ points.

Efficiency Theorem: If there is a small disagreement coefficient $\theta$, the algorithm requires only $O\left(\theta \sqrt{t \log t}\right) + \text{a minimum due to noise.}$
Characterizes known examples where active learning can help. Defined for any set of classifiers $H$ and distribution $D$. 
Disagreement Coefficient

Characterizes known examples where active learning can help. Defined for any set of classifiers $H$ and distribution $D$.

For any $\epsilon$ features $x$ are of interest if there exists a hypothesis $h$:

1. With error rate less than $\epsilon$ larger than the best $h^*$.
2. That disagrees with the best hypothesis, $h^*(x) \neq h(x)$. 
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Disagreement coefficient is $\theta = \max_\epsilon \Pr(\text{interesting}_\epsilon x)$
Disagreement coefficient: examples

Thresholds in $\mathbb{R}$, any data distribution.

$\theta = 2$.

Linear separators through the origin in $\mathbb{R}^d$, uniform data distribution.

$\theta \leq \sqrt{d}$.

Linear separators in $\mathbb{R}^d$, smooth data density bounded away from zero.

$\theta \leq c(\hstar)d$ where $c(\hstar)$ is a constant depending on the target $\hstar$. 

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  \[ \theta \leq c(h^*)d \]
  where $c(h^*)$ is a constant depending on the target $h^*$. 
Decision Tree Experiments

- **MNIST 3s vs 5s**
  - Test error decreases as the number of labels queried increases.
  - Active querying outperforms passive querying.

- **KDDCUP99**
  - Test error decreases as the number of labels queried increases.
  - Active querying outperforms passive querying.

- **KDDCUP99 (close-up)**
  - Detailed view of the test error drop for KDDCUP99.

- **MNIST multi-class (close-up)**
  - Detailed view of the test error drop for MNIST multi-class.
An Approximate IWAL

Let $h(x) = \text{Learn}(S)$.
Let $h'(x) = \text{Learn}_{h(x) \neq y}(S)$.

Claim: If $\text{Learn}$ minimizes error rates, for all $\epsilon > 0$

$$\text{Learn}(S \cup (x, -h(x), t\Delta + \epsilon)) = h'(x)$$

In other words $t\Delta =$ importance weight required to change label for current $x$. 

Using Vowpal Wabbit as base learner, estimate $t \cdot \Delta$ as the number of gradient updates with $x$ required for prediction to switch (from 0 to 1, or from 1 to 0).

e.g. for importance weight-aware square-loss update:

$$\Delta_t = \frac{1}{t \cdot \eta_t \cdot \log \max\{h(x), 1 - h(x)\}}.$$
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* e.g., for importance weight-aware square-loss update:

$$\Delta_t := \frac{1}{t \cdot \eta_t} \cdot \log \frac{\max\{h(x), 1 - h(x)\}}{0.5}$$
active_interactor.cc (in git repository) demonstrates how to implement this protocol.
Fringe Benefits

This approach has **many** nice properties.
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   2. Unlabeled data efficient.
   3. Computationally efficient.
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3. Compatible.
   1. With Online Algorithms
   2. With any optimization-style classification algorithm.
   3. With any Loss function
   4. With supervised learning
   5. With switching learning algorithms (!)
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   5. With switching learning algorithms (!)
4. It works, empirically.
1 Importance weighting
2 Rare Classes
3 Cost Sensitive
How many labels for a rare class?
How many labels for a rare class?

Attenberg & Provost 2010: Search and insertion of labeled rare class examples helps.
Why does search help?

Potentially: exponential improvement in label complexity!
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When does Search help?

Does searching for counterexamples help?
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No!
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No!
When does Search help?

Does searching for counterexamples help?

No!
Counterexample to version space instead!
Theorem: Search for version space counterexample can reduce Label calls exponentially by starting with simple set of classifiers and moving to more complex as they are proved inadequate.
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1. Importance weighting
2. Rare Classes
3. Cost Sensitive
Cost-sensitive multi-class classification

Distribution $D$ over $X \times [0, 1]^k$, where a vector in $[0, 1]^k$ specifies the cost of each of the $k$ choices.

Find a classifier $h : X \rightarrow \{1, \ldots, k\}$ minimizing the expected cost

$$\text{cost}(h, D) = \mathbf{E}_{(x, c) \sim D}[c_h(x)].$$
How to do Active Cost Sensitive Classification?

Cost-sensitive multi-class classification

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Find a classifier $h : X \rightarrow \{1, \ldots, k\}$ minimizing the expected cost

$$\text{cost}(h, D) = \mathbb{E}_{(x, c) \sim D}[c_{h(x)}].$$

Should queries be per-example or per-cost?
Which class costs should be queried?

- Minimum cost
- Cost difference matters
Which class costs should be queried?

Minimum cost < smallest maximum cost
Which class costs should be queried?

1. Minimum cost < smallest maximum cost
2. Cost difference matters
Cost Overlapped Active Learning Results

Theorem: Works efficiently if (1) cost predictors factorize (2) squared loss optimizer is efficient (3) world is IID.
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![Graph showing RCV1-v2 test cost vs number of queries](image)

- Passive
- COAL (1e-1)
- COAL (1e-2)
- COAL (1e-3)
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