

# Active Learning via Reduction To Supervised Classification

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Can a learning algorithm effectively interactively choose which examples to label?

## The Active Learning Setting

Repeatedly:

- 1 Observe unlabeled example  $x$ .
- 2 Make prediction  $\hat{y}$ .
- 3 Asking for label? Yes/no
- 4 If yes, observe label  $y$ .

Goal: Simultaneously minimize the number of mistakes and the number of labels requested.

Good solutions imply more efficient learning *and* a better understanding of how to deal with other forms of interactive learning.

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Start with a pool of unlabeled data

Pick a few points at random and get their labels

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- Fit a classifier to the labels seen so far

- Query the unlabeled point that is closest to the boundary  
(or most uncertain, or most likely to decrease overall uncertainty,...)

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Even with infinitely many labels, converges to a classifier with 5% error instead of the best achievable, 2.5%. *Not consistent!*

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- 3 BDL 2009: The same for other loss functions.
- 4 BHLZ 2010: Yes, given an efficient loss optimization algorithm. This talk.

# Importance Weighted Active Learning via Reduction

$$S = \emptyset$$

While (unlabeled examples remain)

- 1 Receive unlabeled example  $x$ .
- 2 Set  $p = \text{Rejection-Threshold}(x, S)$ .
- 3 If  $U(0, 1) \leq p$ , get label  $y$ , and add  $(x, y, \frac{1}{p})$  to  $S$ .
- 4 Let  $h = \text{Learn}(S)$ .

Consistency: (BDL2009) For all reasonable choices of Rejection-Threshold, the algorithm is consistent.

# What should **Rejection-Threshold** be?

On the  $k$ th unlabeled point, let:

$\hat{e}(h, S) = \frac{1}{k} \sum_{(x,y,i) \in S} i \mathbb{1}(h(x) \neq y) =$  importance weighted error rate.

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Success: (BHLZ2010) If there is a small **disagreement coefficient**  $\theta$ , the algorithm requires only  $O(\theta \sqrt{k \log k})$  + a minimum due to noise (K2006).

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Disagreement coefficient is  $\theta = \max_x \frac{\Pr(\text{interesting}_\epsilon x)}{\epsilon}$   
(See ICML 2009 tutorial for examples)

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- 2 The number of coin flips is minimized:  $\min \sum_{(h,p) \in S} p$ .
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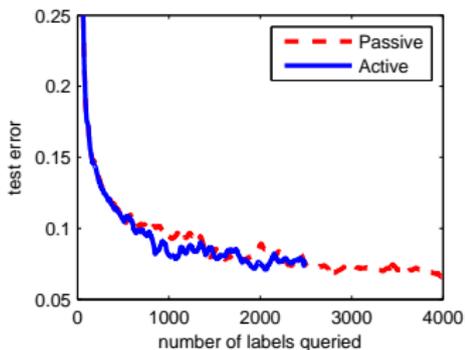
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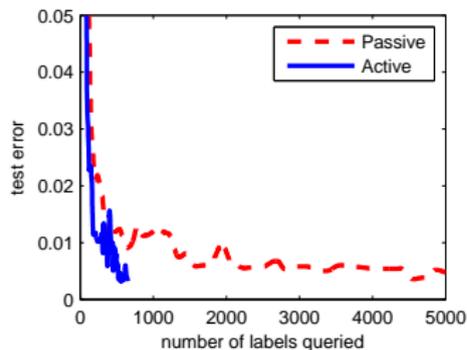
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$p$  too small, implies that condition (1) is violated with a reasonable probability.

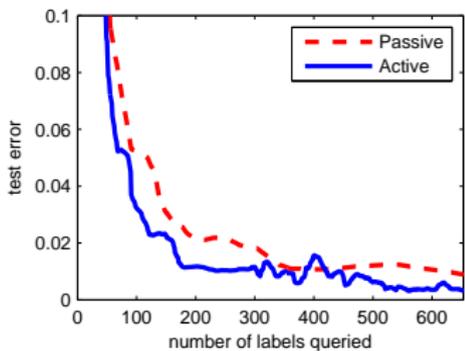
# Decision Tree Experiments



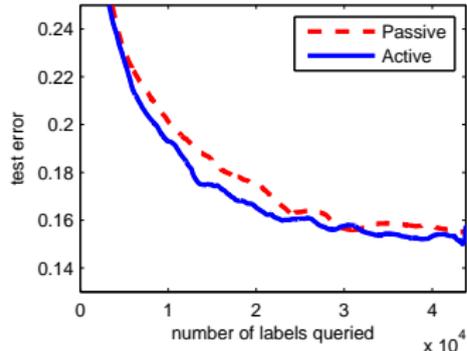
MNIST 3s vs 5s



KDDCUP99

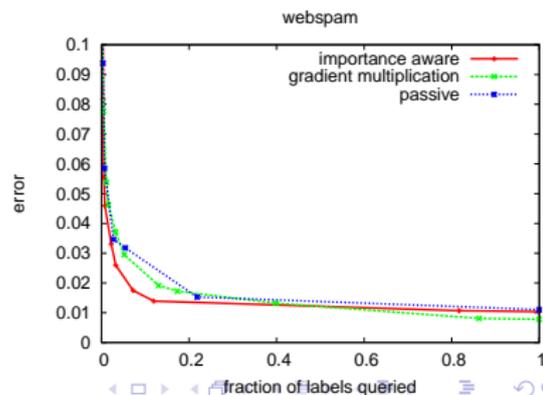
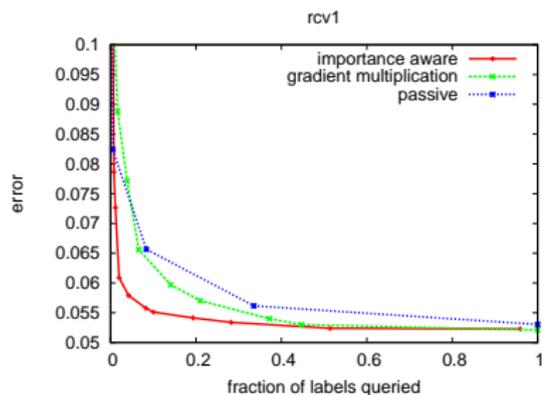
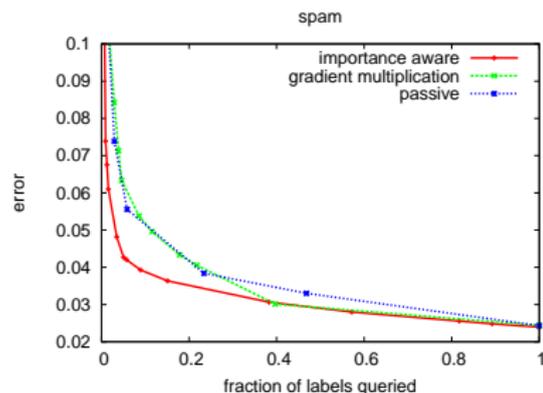
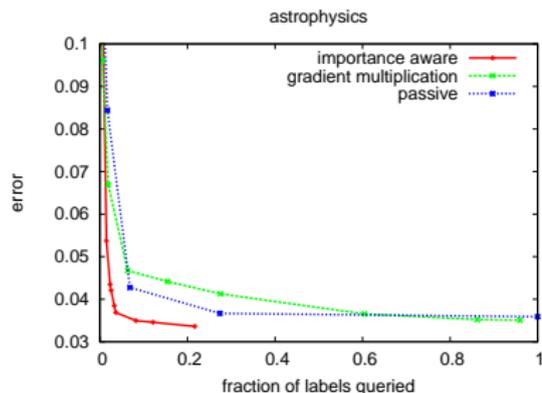


KDDCUP99 (close-up)



MNIST multi-class (close-up)

# Online Linear Learning results (with Nikos)



# Demonstration

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- 9 Allows you to switch learning algorithms later (!)
- 10 Empirically, yields substantial label savings.

Active Learning is only one kind of interactive learning. Does a similar strategy work with other forms of interactive learning?

# Bibliography

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