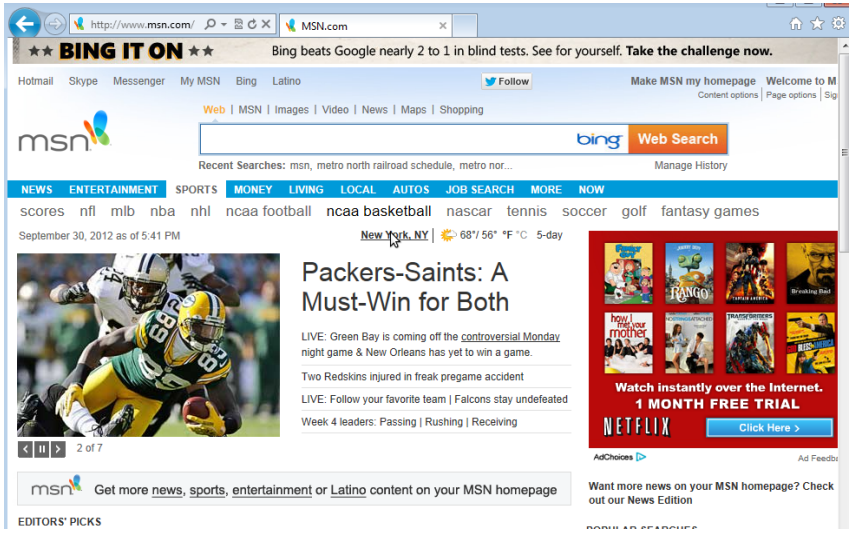
Learning to Interact

John Langford @ Microsoft Research (with help from many)

Slides at: http://hunch.net/~jl/interact.pdf

For demo:
Raw RCV1 CCAT-or-not:
http://hunch.net/~jl/VW_raw.tar.gz
Simple converter: wget http://hunch.net/~jl/cbify.cc
Vowpal Wabbit for learning: http://hunch.net/~vw

Examples of Interactive Learning



Repeatedly:

1. A user comes to Microsoft (with history of previous visits, IP address, data related to an account)
2. Microsoft chooses information to present (urls, ads, news stories)
3. The user reacts to the presented information (clicks on something, clicks, comes back and clicks again,...)

Microsoft wants to interactively choose content and use the observed feedback to improve future content choices.

Another Example: Clinical Decision Making

"Whoa—way too much information."

Repeatedly:

1. A patient comes to a doctor with symptoms, medical history, test results
2. The doctor chooses a treatment
3. The patient responds to it

The doctor wants a policy for choosing targeted treatments for individual patients.

The Contextual Bandit Setting

For $t = 1, \ldots, T$:

1. The world produces some context $x \in X$
2. The learner chooses an action $a \in A$
3. The world reacts with reward $r_a \in [0, 1]$

Goal: Learn a good policy for choosing actions given context.

The Evaluation Problem

Let $\pi : X \to A$ be a policy mapping features to actions. How do we evaluate it?

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Method 1: Deploy algorithm in the world.

Very Expensive!

The "Direct method"

Use past data to learn a reward predictor $\hat{r}(x, a)$, and act according to $\arg\max_a \hat{r}(x, a)$.

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	a_1	a_2
x_1		
x_2		

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Observed

	a_1	a_2
x_1	.8	?
x_2	?	.2

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Observed/Estimated

	a_1	a_2
x_1	.8/.8	?/.5
x_2	?/.5	.2 /.2

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x_1	.8/.8	?/.514
x_2	.3/.3	.2 /.014

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Basic observation 1: Generalization alone is not sufficient.

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Basic observation 2: Exploration is required to succeed.

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Basic observation 3: Prediction errors not controlled exploration.

Outline

Method 3: The Importance Weighting Trick

Let $\pi : X \to A$ be a policy mapping features to actions. How do we evaluate it?

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Let $\pi : X \to A$ be a policy mapping features to actions. How do we evaluate it?

One answer: Collect T exploration samples of the form

$$(x, a, r_a, p_a),$$

where
x = context
a = action
r_a = reward for action
p_a = probability of action a
then evaluate:

$$\text{Value}(\pi) = \text{ Average } \left(\frac{r_a \, \mathbf{1}(\pi(x) = a)}{p_a} \right)$$

The Importance Weighting Trick

Theorem

For all policies π, for all IID data distributions D, Value(π) is an unbiased estimate of the expected reward of π:

$$\mathbf{E}_{(x,\vec{r})\sim D}\left[r_{\pi(x)}\right] = \mathbf{E}[\,\mathrm{Value}(\pi)\,]$$

with deviations bounded by

$$O\left(\frac{1}{\sqrt{T \min_x p_{\pi(x)}}}\right)$$

Proof: [Part 1] $\mathbf{E}_{a\sim p}\left[\frac{r_a \mathbf{1}(\pi(x)=a)}{p_a}\right] = \sum_a p_a \frac{r_a \mathbf{1}(\pi(x)=a)}{p_a} = r_{\pi(x)}$

What if you don't know probabilities?

Suppose p was:

1. misrecorded "We randomized some actions, but then the Business Logic did something else."
2. not recorded "We randomized some scores which had an unclear impact on actions".
3. nonexistent "On Tuesday we did A and on Wednesday B".

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Learn predictor $\hat{p}(a|x)$ on $(x, a)^*$ data.

Define new estimator: $\hat{V}(\pi) = \hat{E}_{x,a,r_a}\left[\frac{r_a I(\pi(x)=a)}{\max\{\tau, \hat{p}(a|x)\}}\right]$ where $\tau =$ small number.

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Theorem: For all IID D, for all policies π with $p(a|x) > \tau$

$$|\text{Value}(\pi) - E\hat{V}(\pi)| \leq \frac{\sqrt{\text{reg}(\hat{p})}}{\tau}$$

where $\text{reg}(\hat{p}) = \mathbf{E}_{x\sim D, a\sim p(a|x)}[(p(a|x) - \hat{p}(a|x))^2]$ = squared loss regret.

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Let $\Delta(a, x) = \hat{r}(a, x) - E_{\vec{r}|x} r_a$ = reward deviation
Let $\delta(a, x) = 1 - \frac{p_a}{\hat{p}_a}$ = probability deviation

Theorem

For all policies π and all $(x, \vec{r})$:

$$|\text{Value'}(\pi) - E_{\vec{r}|x}[r_{\pi(x)}]| \leq |\Delta(\pi(x), x)\delta(\pi(x), x)|$$

The deviations multiply, so deviations < 1 means we win!

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Apply transform to RCV1 dataset.
wget http://hunch.net/~jl/VW_raw.tar.gz
wget http://hunch.net/~jl/cbify.cc
Output format is:
action:cost:probability | features
Example:
1:1:0.5 | tuesday year million short compan vehicl line stat financ commit exchang plan corp subsid credit issu debt pay gold bureau prelimin refin billion telephon time draw basic relat file spokesm reut secur acquir form prospect period interview regist toront resourc barrick ontario qualif bln prospectus convertibl vinc borg arequip
...

How do you train?

1. Learn $\hat{r}(a, x)$.
2. Compute for each x the double-robust estimate for each $a' \in \{1, ..., K\}$:

$$\frac{(r - \hat{r}(a, x))I(a' = a)}{p(a|x)} + \hat{r}(a', x)$$

3. Learn π using a cost-sensitive classifier. We'll use Vowpal Wabbit: `http://hunch.net/~vw`

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```
vw –cb 2 –cb_type dr rcv1.train.txt.gz -c –ngram 2 –skips 4 -b 24 -l 0.25
  Progressive 0/1 loss: 0.04582
vw –cb 2 –cb_type ips rcv1.train.txt.gz -c –ngram 2 –skips 4 -b 24 -l 0.125
  Progressive 0/1 loss: 0.05065
vw –cb 2 –cb_type dm rcv1.train.txt.gz -c –ngram 2 –skips 4 -b 24 -l 0.125
  Progressive 0/1 loss: 0.04679
```

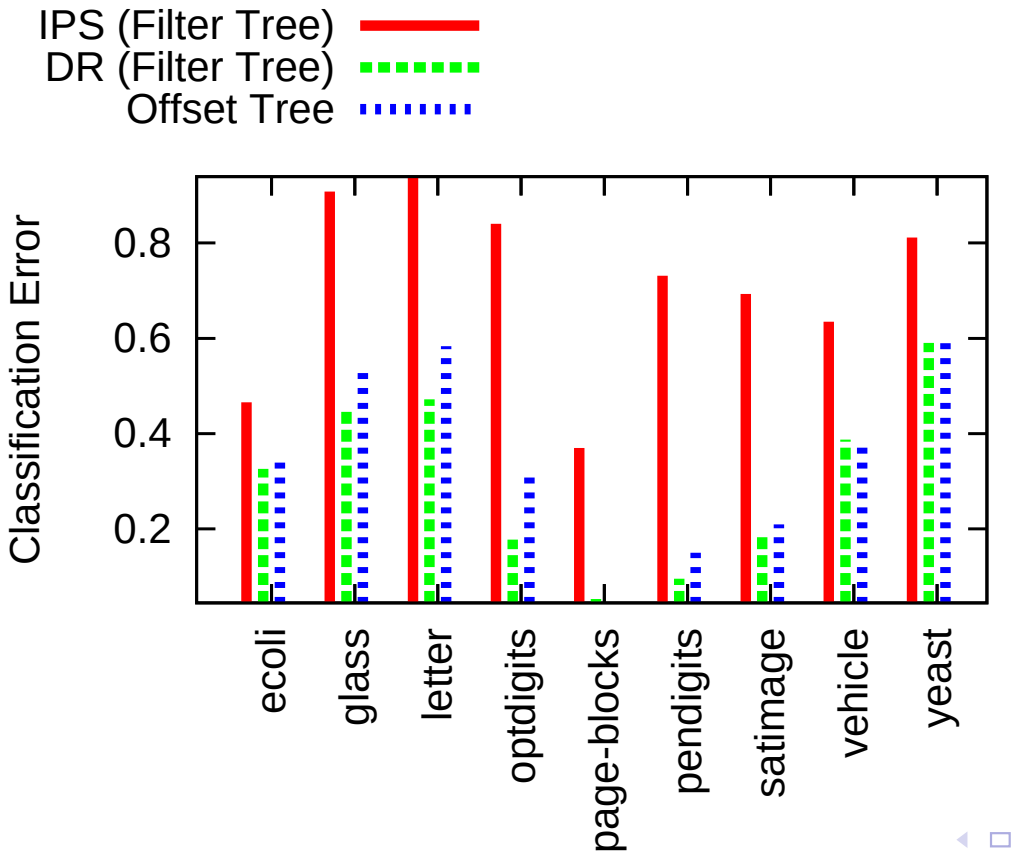
Experimental Results

IPS = Inverse probability

DR = Doubly Robust, with $\hat{r}(a, x) = w_a \cdot x$

Filter Tree = Cost Sensitive Multiclass classifier

Offset Tree = Earlier method for CB learning with same representation



Summary of methods

1. Deployment. Aka A/B testing. Gold standard for measurement and cost.
2. Direct Method. Often used by people who don't know what they are doing. Some value when used in conjunction with careful exploration.
3. Inverse probability. Unbiased, but possibly high variance.
4. Inverse propensity score. For when you don't know or don't trust recorded probabilities. Debugging tool that gives hints, but caution is in order.
5. Offset Tree. (not discussed) Only known logarithmic time method.
6. Double robust. Best known offline method. Unbiased + reduced variance.

Reminder: Contextual Bandit Setting

For $t = 1, \ldots, T$:

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Goal: Learn a good policy for choosing actions given context.

What does learning mean? Efficiently competing with some large reference class of policies $\Pi = \{\pi : X \to A\}$:

$$\text{Regret} = \max_{\pi \in \Pi} \text{average}_t(r_{\pi(x)} - r_a)$$

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We discuss Randomized here.

1. There are no good deterministic exploration algorithms in this setting.
2. Supports off-policy evaluation. (See first half.)
3. Randomize = robust to delayed updates, which are very common in practice.

Outline

1. Using Exploration
    1. Problem Definition
    2. Direct Method fails
    3. Importance Weighting
    4. Missing Probabilities
    5. Doubly Robust
2. Doing Exploration
    1. Exploration First
    2. ϵ-Greedy
    3. epoch Greedy
    4. Policy Elimination
    5. Thompson Sampling

Explore τ then Follow the Leader (Explore-τ)

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Initially, $h = \emptyset$

For the first τ rounds

1. Observe x.
2. Choose a uniform randomly.
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For the next T rounds, use empirical best.

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Suppose all examples are drawn from a fixed distribution $D(x, \vec{r})$.

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At optimal τ? $O\left((\frac{|A| \ln |\Pi|}{T})^{1/3}\right)$

What does this mean?

1. +Easiest approach: offline prerecorded exploration can feed into any learning algorithm. See first half.
2. -Doesn't adapt when world changes.
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Can we do better?

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1. Observe x.
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   1. Choose learned a
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Theorem: Epoch Greedy has regret $O\left(\left(\frac{|A| \ln |\Pi|}{T}\right)^{1/3}\right)$ with high probability.

Autotuning!

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Is it possible to do better?

	Supervised	τ-first/ϵ-greedy/epoch-greedy
Regret	$O\left(\left(\frac{\ln \lvert\Pi\rvert}{T}\right)^{\frac{1}{2}}\right)$	$O\left(\left(\frac{\lvert A\rvert \ln \lvert\Pi\rvert}{T}\right)^{\frac{1}{3}}\right)$

Better 1: Policy Elimination

Policy_Elimination

Let $\Pi_0 = \Pi$ and $\mu_t = 1/\sqrt{Kt}$ and $\eta_t(\pi)$ =empirical reward
For each $t = 1, 2, \ldots$

1. Choose distribution P over Π_{t-1} s.t. for every remaining policy π, the expected variance of a value estimate is small.
2. observe x
3. Let $p(a)$ = fraction of P choosing a given x.
4. Choose $a \sim p$ and observe reward r
5. Let Π_t = remaining near empirical best policies.

Theorem: With high probability Policy_Elimination has regret

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1. Choose distribution P over Π_{t-1} s.t. $\forall\ \pi \in \Pi_{t-1}$:
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3. Let $p(a) = (1 - K\mu_t)\Pr_{\pi \sim P}(\pi(x) = a) + \mu_t$
4. Choose $a \sim p$ and observe reward r
5. Let $\Pi_t = \{\pi \in \Pi_{t-1} : \eta_t(\pi) \geq \max_{\pi' \in \Pi_{t-1}} \eta_t(\pi') - K\mu_t\}$

Theorem: With high probability Policy_Elimination has regret

$$O\left(\sqrt{\frac{|A| \ln |\Pi|}{T}}\right)$$

What does this mean?

1. -Doesn't adapt when world changes.
2. ++Much more efficient exploration. Only efficient in special cases.
3. - -Much Harder Approach: Need to keep track of policies, which is often intractable.

What does this mean?

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2. ++Much more efficient exploration. Only efficient in special cases.
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Adapting algorithms exist (EXP4).
More efficient versions exist (RUCB), but not yet efficient enough.

Can you do better?

Can you do better?

Not in general.

Theorem: For all algorithms, there exists problems imposing regret:

$$\tilde{\Omega}\left(\sqrt{\frac{|A|\ln|\Pi|}{T}}\right)$$

Better 2: Thompson Sampling

Always maintain a Bayesian posterior over policies.
On each round sample policy from posterior, and act according to it.

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On each round sample policy from posterior, and act according to it.
An efficient special case: Gaussian Posterior.

Thompson Sampling

Let w = mean 0 multivariate gaussian.
For each $t = 1, 2, \ldots$

1. Draw $w' \sim w$
2. Observe x
3. Choose $a = \max_{a'} w' x_{a'}$
4. Observe reward r.
5. Bayesian update w with (x, a, r).

What does it mean?

1. +Efficient special cases for Gaussian posteriors.
2. +Known to work well empirically sometimes.
3. -Not robust to model misspecification: $\tilde{\Omega}\left(\frac{|\Pi|}{T}\right)$ regret.

The current state

Starter	
Baseline	
Purring	
Shiny	
Something to try	

The current state

Explore-τ	Simplest Possible
Baseline	
Purring	
Shiny	
Something to try	

The current state

Explore-τ	Simplest Possible
ϵ-Greedy	Simplest Adaptive
Purring	
Shiny	
Something to try	

The current state

Explore-τ	Simplest Possible
ϵ-Greedy	Simplest Adaptive
Epoch Greedy	Unequivocal Improvement
Shiny	
Something to try	

The current state

Explore-τ	Simplest Possible
ϵ-Greedy	Simplest Adaptive
Epoch Greedy	Unequivocal Improvement
Policy Elimination	Optimal Impractical
Something to try	

The current state

Explore-τ	Simplest Possible
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Thompson Sampling	Sometimes Excellent

The current state

Explore-τ	Simplest Possible
ϵ-Greedy	Simplest Adaptive
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Policy Elimination	Optimal Impractical
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You can see the edge of the understood world here. We hope to see further soon.

Further discussion: http://hunch.net

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